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A THEORY OF VOTING GAMES

AND

THE POWERIND PROGRAM

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1. VOTING GAMES

A **simple game** is defined as a couple (**N**,**W**) where **N** is a finite set of **players** or **voters** (we put **N**={1,…,*n*} for convenience) and **W** is a family of subsets of **N** which satisfies axioms A1 and A2. The subsets of **N** will be referred to as **coalitions**. The elements of **W** will be called **winning coalitions**.

A1: **W** $\neq \varnothing$ (there is at least one winning coalition).

A2: *If* $C \in$ W *and* $C \subset C'$ *then* $C' \in$ **W** (if further players join a winning coalition then a \`supercoalition' of *C* so obtained is also winning).

A1 and A2 imply that *N*, or the **grand coalition**, is winning. A simple game is called **proper** if the following axiom also holds.

A3: *If* $C \in$ **W**, *then* $N\text{-}C \notin$ **W** (the complement of a winning coalition is not a winning coalition).

Proper simple games will also be called **voting games**. If axiom A4 is met, a voting game is said to be **strong**.

A4: *For any* $C \subset$ **N**, $C \in$ **W** *or* $N\text{-}C \in$ **W**.

A coalition *C* is called **losing** if $N\text{-}C \in$ **W**. This definition and A3 imply that: (i) for any coalition *C*, *C* is winning if and only if *N*-*C* is losing; (ii) a subcoalition of a losing coalition is losing.

A coalition that is neither winning nor losing is called **blocking**. The complement of a blocking coalition is also blocking. A voting game is strong if and only there is no blocking coalition.

Note: The term \`losing' is often referred to coalitions that are not in **W**. Any *C* such that $N\text{-}C \notin$ **W** is then called \`blocking'. Our terminology does not follow this common usage.

The set of *all* coalitions which has $a=2^n$ elements ($n=|$**N**$|$ stands for the number of voters) is the union of 3 pairwise disjoint classes:

W – the set of winning coalitions

B – the set of blocking coalitions

L – the set of losing coalitions

The order **W**>**B**>**L** in which the three classes are listed above will be called the **power order**. We say that winning coalitions are **more powerful** than blocking coalitions which in turn are more powerful than losing coalitions.

A winning or blocking coalition *C* is called **minimal** if no proper subcoalition of C is equally powerful as C.

The number l=|**L**| of losing coalitions equals the number w=|**W**| of winning coalitions. Hence a=2w+b where b=|**B**| stands for the number of blocking coalitions. As a consequence, any voting game has at most ½a=2^{n-1} winning coalitions (notice that all strong games have the same number of winning coalitions). Hence the maximum **efficiency** w/a of a voting game equals ½. The **consensus game** which has exactly one winning coalition (the grand coalition N) is least efficient.

The **union** and **intersection** of two simple games (**N**,**W**) and (**N**,**W'**) are defined as (**N**,**W**∪**W'**) and (**N**,**W**∩**W'**), respectively. The union of two simple games is a simple game. The union of two voting games is a voting game if and only if $C \cap C' \neq \varnothing$ for any $C \in$ **W** and any $C' \in$ **W'**. The intersection of two voting games is always a voting game.

Let C be a nonempty subset of **N**. The **voting game generated by** C is defined as a game whose set of winning coalitions consists of all supercoalitions of C. In this game, all subsets of N-C are losing coalitions. The game is not strong except for the case C=N (every blocking coalition is a union of a nonempty subset of C and a nonempty subset of N-C).

Every voting game is the union of voting games generated by all minimal winning coalitions.

We say that (**N**,**W**) is a **subgame** of (**N**,**W'**), or (**N**,**W'**) is an **extension** of (**N**,**W**) if **W** ⊂ **W'**. Any voting game is a subgame of a strong game. To prove this, take a game (**N**,**W**) that is not strong, that is, it has a blocking coalition C. Then, for any $C' \in$ **W**, $C \cap C' \neq \varnothing$ (if $C \cap C' = \varnothing$, then C ⊂ N-$C' \in$ **L**, so that $C \in$ **L**). Hence the union of (**N**,**W**) and of the game generated by C is a voting game being an extension of (**N**,**W**). The extending process can be continued until a strong game is obtained. Strong games are **maximal elements** in the set G(**N**) of all voting games over **N**. The subgame relation is a **partial order** in G(**N**). The consensus game is the only **minimal element** in G(**N**).

Two voting games (**N**,**W**) and (**N**,**W'**) are said to be **isomorphic** if there is a 1-1 mapping T of **N** onto **N** such that $T(C) \in$ **W'** if and only if $C \in$ **W**. Any function which assigns numbers to games in G(**N**) will be called a **structural parameter of a game** if it assumes the same value for any two isomorphic games. A **local structural parameter** is defined as a mapping f of **N** such that $f(i)$=$f(j)$ for any two players i and j such that i=$T(j)$ for some **automorphism** T of (**N**,**W**).

Let **W**(i) denote the set of winning coalitions containing a player i. Since **W**($T(i)$)=**W**(i) for any automorphism T, $w(i)$=|**W**(i)| is a local structural parameter. It is the simplest and least refined measure of **voting power**. Powerful voters are those whose cooperation is needed to form winning coalitions.

A player that is not a member of any minimal winning coalition has no power at all; it is called a **dummy**. We call a **vetoer** any player being a member of all minimal winning coalitions. The term **dictator** will be referred to any i such that $\{i\} \in$ **W**. Clearly, a voting game may have only one dictator. If there is a dictator, then all other players are dummies.

A voting game (**N**,**W**) is said to be **symmetric** if for any two players i and j there exists an automorphism T such that i=$T(j)$. In a symmetric game, $w(i)$=$w(j)$ for any i, j.

2. POWER INDICES

The number $wm(i)$ of minimal winning coalitions containing a player i is a more refined measure of power, yet too many coalitions in which i's membership is critical are skipped if i's power is calculated in such a way.

Let i be a voter in C. Voter i is called a **swing member** of C if C is more powerful than C-{i}. Clearly, no losing coalition C may have a swing voter because C-{i} is also losing. If C is winning, then a voter i is a swing member of C if and only if C-{i} is blocking or losing. If C is blocking, then i is a swing member of C if and only if C-{i} losing.

A winning or blocking coalition C is minimal if and only if every player in C is a swing voter. Let $s(C)$ stand for the number of swing members of C. C is called **vulnerable** if $s(C)>0$. C is minimal if and only if $s(C)=|C|$.

Note: Vulnerable coalitions are often called `minimal' in which case the term `strictly minimal' is used to denote minimal coalitions.

Let $ws(i)$ be the number of winning coalitions containing i as a swing voter, and $wv(i)$ be the number of vulnerable winning coalitions containing i. Clearly,

$$w(i) \geq wv(i) \geq ws(i) \geq wm(i).$$

All four quantities can be used to characterize the winning power of player i. However, $w(i)$ can be disposed of altogether because

$$w(i)=½(ws(i)+w)$$

This formula can be derived from two equations

$$w=w(i)+w^*(i),\ w(i)=ws(i)+ws^*(i)$$

where $w^*(i)$ is the number of elements of $\mathbf{W}^*(i)=\{C\in\mathbf{W}\colon i\notin C\}$ and $ws^*(i)$ is the number of elements of $\boldsymbol{Ws}^*(i)=\{C\in\mathbf{W}\colon i\in C, C-\{i\}\in\mathbf{W}\}$. Now notice that $C\rightarrow C\cup\{i\}$ is a 1-1 mapping of $\mathbf{W}^*(i)$ onto $\boldsymbol{Ws}^*(i)$. Therefore, $w^*(i)=ws^*(i)$ and the two equations can be reduced to $w(i)=ws(i)+w-w(i)$, or $2w(i)=ws(i)+w$.

By defining 4 analogous sets of blocking coalitions containing i, we obtain 4 parameters

$$b(i) \geq bv(i) \geq bs(i) \geq bm(i).$$

The first of them cannot be used to measure the **blocking power** of a player because $b(i)$, just like the number $a(i)=2^{n-1}$ of all coalitions containing i, is identical for every i. To prove that

$$b(i)=2^{n-1}-w$$

notice first that $C\rightarrow N-C$ is a 1-1 mapping of the set $\mathbf{L}(i)$ of losing coalitions containing i onto $\mathbf{W}^*$(i). Hence, $l(i)=w^*(i)$. Now observe that

$$w=w(i)+w^*(i)=w(i)+l(i),\ a(i)=w(i)+b(i)+l(i).$$

Therefore, $b(i)=a(i)-(w(i)+l(i))=2^{n-1}-w$

Three measures of **total voting power** are defined by the formulas

$$tv(i)=wv(i)+bv(i),\ ts(i)=ws(i)+bs(i),\ tm(i)=wm(i)+bm(i).$$

There are many ways of normalizing power coefficients. By dividing *ws*(*i*) by *w* we get **Coleman's Preventive Power Index** (equal to 1 if and only if *i* is a vetoer). The quotient $ws(i)/2^{n-1}$, known as the **Banzhaf Absolute Power Index** or **Penrose Index**, can be computed by multiplying Coleman's index by **relative efficiency** $w/2^{n-1}$. The latter is obtained from efficiency $w/2^n$ (named `collectivity index' by Coleman) by dividing it by its theoretical maximum, or ½.

Relative normalized power indices are those obtained by adding up the values of an absolute measure over *n* players and dividing each value by the sum. Then the index values not only lie in the [0,1] interval, but they add up to 1 so that one can compare **power distributions** across all games with the same set of players. The level of power concentration can be assessed by means of the standard deviation of the values of an index (their arithmetic mean always equals 1/*n*).

The **Banzhaf-Coleman Power Index** is defined as the relative normalized number of winning coalitions containing a voter *i* as a swing member. Symbolically,

$$\beta(i) = ws(i)/\sum_{j=1}^{n} ws(j)$$

Every winning coalition *C* containing a swinger *i* contributes the same value 1 to *i*'s total absolute score regardless of whether *i* is the unique voter in *C* whose presence is essential for *C* to be winning or *C* has more such members. The portion of *i*'s overall power gained from being a swing member in *C* is defined as 1/*s*(*C*), or the inverse of the number of swingers in *C*. The relative normalized index based on 1/*s*(*C*) is known as the **Johnston Power Index**. If the summation is restricted to minimal winning coalitions ($1/s(C)=1/|C|$), we get the **Deegan-Packel Power Index**.

Lastly, we define the **Shapley-Shubik Power Index**, the coefficient most often used along with the Banzhaf-Coleman Index. The Shapley-Shubik index is a special case of the Shapley value, which is a kind of solution proposed by Shapley for any game in characteristic function form (see any game theory handbook). The definition given below does not make reference to the general case.

Consider all possible **orderings** of n players. Each ordering is one of *n*! ways in which the grand coalition can be built up by enlarging an already formed coalition by one player at each step. Any sequence $i_1,\ldots,i_n$ contains only one player i_j such that the coalition $\{i_1,\ldots,i_n\}$ is winning, but the coalition of the players preceding i_j in the sequence is not. The value of the Shapley-Shubik index for voter *i* is the probability that *i* is the player whose consent to join an initial sequence causes the transition from a non-winning to winning coalition. One can show that this probability equals the sum of weights of all coalitions containing voter i as a swinger, the weight of C being related to $k=|C|$ by the formula $((k-1)!(n-k)!)/n!$. Note that the values of the Shapley-Shubik index are already normalized, being defined as the probabilities of pairwise disjoint events the union of which is the set of all permutations.

3. WEIGHTED VOTING GAMES

Given a sequence of numerical **weights** $p_1,\ldots,p_n$ assigned to players $1,\ldots,n$ and a **quota** q, the set of winning coalitions can be defined by the condition: $C \in \mathbf{W}$ if and only if $p(C) \geq q$ where $p(C$. or the **weight of a coalition** C, is the sum of p_i over all $i \in C$.

It is assumed that weights are nonnegative real numbers such that $p(N)>0$ and q is a number such that $q > ½p(N)$ and $q \leq p(N)$. These assumptions imply that axioms A1–A3 are met so that ($\mathbf{N}$,$\mathbf{W}$) is, in fact, a voting game.

Since $p(N\text{-}C)=p(N)-p(C)$, a coalition C is losing if and only if $p(C) \leq p(N)-q$, and blocking if and only if $p(N)-q < p(C) < q$.

Can every voting game be represented as a weighted voting game? The following example shows that the answer to this question is negative. Let N={1,2,3,4,5}. Consider the union of two voting games generated by C_1={1,2,3} and C_2={3,4,5}, respectively. The remaining winning coalitions are C_3={1,2,3,4}, C_4={1,2,3,5}, C_5={1,3,4,5}, C_6={2,3,4,5}, $C_7=N$. Suppose that there exist weights $p_1,\ldots,p_5$ and a quota q such that $C \in$ W if and only if $p(C) \geq q$. By applying this condition to C_1 and C_2 we get the inequalities $p_1+p_2+p_3 \geq q$ and $p_3+p_4+p_5 > q$ which imply that $2p_3+p_1+p_2+p_4+p_5 \geq 2q$. Hence $p_3+½(p_1+p_2+p_4+p_5) \geq q$. One of two numbers p_1+p_4 and p_2+p_5 must be greater than or equal to the other. If p_1+p_4 is the greater number, then $p_3+p_1+p_4 \geq p_3+½(p_1+p_2+p_4+p_5) \geq q$ so that the subset {1,3,4} would have to be a winning coalition, but it is a blocking coalition.

The simplest weighted voting game is the **one voter-one vote simple majority game** in which p_i=1 for all i and q is the smallest integer greater than ½n. If n is odd, the simple majority game is strong. For n even, every coalition of ½n players is blocking. This game can be extended to a strong game by endowing a player, say, player 1, with the right to resolve a tie, that is, a set C of ½n players becomes a winning coalition if and only if player 1 is in C. Equivalently, put p_1'=2, p_i'=1 for $i=2,\ldots,n$, and set q to ½n+1. Note that the extended game is not symmetric. In the non-symmetric game just defined, voter 1, or the `chairman', has more power than his colleagues. For n=4 there are 8 winning coalitions: C_1={1,2}, C_2={1,3}, C_3={1,4}, C_4={2,3,4}, C_5={1,2,3}, C_6={1,2,4}, C_7={1,3,4}, C_8={1,2,3,4}. Player 1 is in 7 out of 8 winning coalitions, in 6 out of 7 being a swing member. Each of 3 remaining players is in 5 winning coalitions, in 2 being a swinger (player 2 in C_1 and C_4). The sum of swing winning coalitions equals 6+2+2+2=12 so that the value of the Banzhaf-Coleman index is 6/12=½ for the `chairman' and 2/12=1/6 for `ordinary' members of the 4-person group.

Politicians and commentators who are not familiar with the theory of voting games tend to believe that the power of a player in a weighted voting game is equal, directly proportional to or at least strongly correlated with the player's weight. Let us begin from the key fact: if $p_i=p_j$, then the transposition of i and j is an automorphism of any voting game dictated by the given collection of weights and any q. As a consequence, i and j are structurally interchangeable and all power indices assume the same value for these two players.

If $p_i \geq p_j$, then the number of winning coalitions containing voter *i* (and that of those in which *i* is a swinger) is greater than or equal to the respective number for voter *i*. However, if only minimal winning coalitions are used in computing a power index, then the ordering of the players with respect to decreasing values of the index need not be consistent with their ranking with respect to decreasing weights. In addition, even if power is quantified by an index based on counting swings across all winning coalitions, the distances between the index values need not agree in size with the respective distances between absolute or relative weights. If the weight of a player *i* exceeds or equals the quota, then such a game coincides with the dictatorial game generated by {*i*} no matter how large is p_i; then the Banzhaf-Coleman index equals 1 for player *i* and 0 for all other players.

Although all power indices discussed earlier in this note are defined for abstract voting games, one can design an index which applies to weighted voting games only: the number of minimal weight coalitions containing a given player. *C* is a **minimal weight coalition** if $p(C)$ is equal to the minimum of $p(C')$ across all *C*' such that $p(C') \geq q$.

4. VOTING IN THE EUROPEAN COUNCIL

I wrote the first version of this program to calculate two most often used power coefficients (Banzhaf-Coleman and Shapley-Shubik indices) for the voting games which formalize the way in which decisions will be made by the **European Council** according to the rules agreed-on in Nice. The **Nice Treaty** introduced a new voting system to be used after the EU is enlarged by 10 new member states. As soon as the **Accession Treaty** had been signed and ratified, old and new EU states had to take a stance on the **EU Constitution draft** in which the Nice Treaty game was replaced with another game. The **EU Constitution double majority game** can be formally defined as the intersection of the one voter-one vote simple majority game and a weighted voting game with relative population size assigned as weight to each EU state and *q* set to 60%.

The **Nice Treaty game** is an intersection of three games, the simple majority game, the population weighted game with *q*=62%, and the game defined by assigning to every country a certain quantity of votes (ranging from 29 to 3) and setting the decision threshold to a number close to ¾ of all allocated votes (321 votes for EU-25). The third game is in line with the long practice of defining weights on the basis of a political agreement. In particular, since 1973 (when UK joined the EU) all big and small European players approved of the principle that 4 strongest of them (Germany, France, UK, and Italy) should have equal influence on EU decisions. The EU Constitution game has been criticized by Poland and Spain not only because the values of the power indices for the two countries are lower than those obtained for the Nice game. What is contested even stronger is an attempt to radically change the way in which the European political system has worked until now. The use of population weights means that the principle of political balance within the Big Four is no longer accepted as the cornerstone of united Europe. Germany with 82 million people now becomes more powerful than France, UK, and Italy, as the Banzhaf-Coleman index equals .133 for this country compared to .095 for UK and France. Germany benefits from the Constitution in absolute terms as well, by gaining more power not only than it has got under the Nice Treaty (.084) but compared to the amount it had in EU-15 (.112).

5. HOW TO USE POWERIND

You can use POWERIND not only to compute power indices for the two games at issue, but for any other game you would suggest to your government as a compromise solution to be defended in negotiations. Before you run POWERIND, you should prepare an **input file** properly named and structured. The filename must have up to 8 characters and no extension (letters A-X and figures 0-9 are accepted only), e.g., EU15, EU25, EU25NICE (the names of input files attached with the program).

An input file should be an ascii text file structured as a sequence of *n+1* lines, with *n*, or the **number of players**, placed in the first line as a single field. The program requires that *n* be an integer in the range from 2 to 27. 27 is the number of EU member states after the expected accession of Romania and Bulgaria.

Lines from 2nd to (n+1)th should contain three fields separated with commas. A **player's name**, given first, must be a string of at most 9 characters placed within quotes, e.g. "Poland   ". Use solely ascii characters ranging from #32 (space) to #126 save for #34 ("). If you put in an empty string ("") or a string of spaces, the program will assign to a player the default name of the form `Player ##'.

The 2nd data item must be an integer written in decimal notation with at most 3 figures. It is the **number of votes** allocated to a player. The input file EU25NICE has here the Nice Treaty weights assigned to 25 states (Germany 29, etc.). The EU25 file gives alternative weights which also add up to 321. These come from the article published in a leading Polish nationwide daily (*Rzeczpospolita*, January 30, 2004) by W. Słomczyński and K. Życzkowski, two Jagiellonian University professors (Faculty of Mathematics and Physics). They proposed that the number of votes assigned to each EU member state for voting in the EU Council be proportional to the **square root of the country's population**. The authors claim that if the weights meet this condition, then all citizens of united Europe will have the same influence on EU decisions.

The content of the E25 file is displayed below. The data field which follows the number of votes is a **country's population** in thousands.

```
25
"Germany  ",33,82357
"UK       ",28,59760
"France   ",28,59191
"Italy    ",28,57948
"Spain    ",23,40266
"Poland   ",23,38641
"Nethrlnds",15,16044
"Belgium  ",12,10264
"Czech R. ",12,10224
"Hungary  ",12,10188
"Portugal ",12,10024
"Greece   ",12,10020
"Sweden   ",11, 8833
"Austria  ",10, 8075
"Slovakia ", 9, 5380
"Denmark  ", 9, 5333
"Finland  ", 8, 5188
"Ireland  ", 7, 3839
"Lithuania", 7, 3481
"Latvia   ", 6, 2355
"Slovenia ", 5, 1992
"Estonia  ", 4, 1364
"Cyprus   ", 3,  756
"Luxemburg", 2,  441
"Malta    ", 2,  395
```

The population data given above comes from the latest (2003) edition of the *Yearly of International Statistics*, an official publication of GUS (Polish national statistical office).

It is required that the 3rd field be in every input file and have an integer format with at most 5 decimal digits. If you're not going to use the population data, put the same figure, say, 0, in each line. If at least two distinct values are found, population weights are processed so as to obtain 3-digit normalized weights. Since the total population of EU-25 is equal to 452359, Germany's normalized weight equals (82357/452359)*1000=182.06 which becomes 182 as weights are always rounded (*k*+.5→*k*+1). Moreover, a special procedure is used to adjust the weights so that they always add up to 1000. 501 is the lowest acceptable quota for defining a voting game with 3-digit normalized weights.

To define a voting game based on a given allocation of votes, you must specify an integer quota. Under the total of 321 votes allocated in the Nice Treaty, the lowest quota equals 116. If you need to analyze a game with a decision threshold given as a fraction, say, ⅔ or 75% you have to employ a calculator to determine the quota to be passed to the program (231*(2/3)=154, 231*.75=173.25, which can be rounded to 173 or 174).

When an input file is read in by POWERIND, you are asked to construct a voting game you want to examine. Your game can be a **single weighted voting game** or an **intersection of 2 or 3 weighted voting games**. You will be prompted to select or skip each of three types of games.

(1) One voter - one vote weighted voting games

(2) Weighted voting games with a `political vote allocation'

(3) Weighted voting games with normalized `population weights'

For each type selected, you have to specify a quota. For example, consider the voting game proposed with an obvious intention to modify the EU Constitution game so as to slightly reduce Germany's power without a loss in efficiency. The **double qualified majority game 55%+55%** with 25 players is an intersection of two games: the one voter – one vote game with quota .55*25=13.75→14 and the population weighted game with quota 550. Thus, to construct this game, select a game of type 1 with q_1=14, skip type 2, and select a type 3 game with q_3=550.

Similarly, to examine the original EU Constitution game, choose games of type 1 and 3 with q_1=13 and q_3=600. The 14/-/550 game is slightly more efficient (*w*/*a*=.23) than the 13/-/600 game (*w*/*a*=.225). The comparison of the Banzhaf-Coleman index values for Germany for the 13/-/600 game and the 14/-/550 game (.133 vs. .941) shows that the latter game is indeed a compromise proposition.

To see the power distribution for the Nice Treaty game, load EU25NICE and select 3 weighted voting games with q_1=13, q_2=241 (75% of 321), q_3=620. Low efficiency (.02) of the Nice Treaty game results from a too high quota in game 2 (if q_2=199=62%, then *w*/*a*=.15) and from the game's complexity (if game 2 is skipped, then *w*/*a*=.20).

This `triple majority game' can be simplified by skipping one of 3 component games. If game 1 is kept (to stress that all EU members are `equal'), then there is still a choice between game 2 with political weights and game 3 with population weights. Słomczyński and Życzkowski resolved the dilemma by defining weights that don't depart too far from the overtly political Nice Treaty weights, yet they are derived from a **mathematical model** which relates – in a nonlinear manner – the number of votes granted to each state to its number of citizens. Thus, the population structure does matter as postulated by the supporters of the EU Constitution. Słomczyński and Życzkowski have also provided a mathematical justification for the quota to be used with `square root weights'. They labeled their game **P-62** where P stands for Penrose (his 1946 paper has long been ignored by the EU experts) and 62% is the decision threshold given as a fraction. If the total of 321 votes allocated by the Nice Treaty among 25 states is left unchanged, then 62% translates into 199 votes.

To examine the P-62 game, load the EU25 input file, skip games of type 1 and 3, and select game 2 with q=199. To test the program, use the EU15 file with political weights currently used by EU-15.

Results are displayed on screen and appended to the **output file** with extension .OUT. If you hit Enter when prompted to type an output file name, the program will adjoin .OUT to the input file name (EU15.OUT will be the **default output file name** for input file named EU15).

The **Power Analysis Menu** allows you to examine winning and blocking coalitions or skip either coalition category during the inspection of all subsets of ***N***. It takes much more time to process both categories (option 0). Thus, for n>15, begin from analyzing winning coalitions only (option 1).

The **coalition statistics** and **game efficiency** is the first result shown when the inspection is over. You will see the number of all coalitions (a), winning (w), losing (l=w), and blocking (b). More specific data (vw=vulnerable winning, mw=minimal winning, vb=vulnerable blocking, mb=minimal blocking) are also given according to the ordered scope of analysis. Next, there goes the data on how many coalitions of a given type every player is a member of. If n>15, this data is not displayed on screen, but it can be saved to the output file.

Values of **4 relative normalized power indices** (Banzhaf-Coleman, Shapley-Shubik, Johnston, and Deegan-Packel) are displayed in turn, multiplied by 10000, rounded, and adjusted to add up exactly to 10000. The Banzhaf Absolute Power Index and Coleman's Preventive Power Index are also shown.

You can also apply POWERIND to many other games, not only those to be played by the EU Council, in particular, to all games where political groupings are players and numbers of MPs they have are their weights.

Having a fast computer is a necessity. My Pentium III 667 Mhz needs some 30 minutes to process winning coalitions in a 25-player game. I am not a professional programmer. **Quick Basic 4.5** which I still use is a pretty old-fashioned tool with a rather poor compiler. However, if you share my disgust at Windows, you will enjoy my program.

Feel free to pass it to everyone interested in voting games. If you use POWERIND to generate data to be published, please, mark this with a reference to:

Tadeusz Sozański. *The POWERIND program*. April 2004.

Http://www.cyf-kr.edu.pl/ ~ussozans/.

Send comments to my email address: ussozans@cyf-kr.edu.pl.

6. A PERSONAL STORY

My current preoccupation with voting games dates back to Spring 2003. For me - it was the time when I completed chapter 5 of my book (*The Mathematics of Exchange Networks*, in process), the chapter that deals with the theory of *n*-person games in characteristic function form. For Poland - it was the time to decide in a national referendum on whether to ratify the Accession Treaty. To persuade my `eurosceptic' wife to vote `yes' rather than `no', I said that the Nice Treaty made Poland and Spain nearly as strong players as the Big Four so that to be inside the EU is better than to be outside.

> *Why do you think so?* - she asked me. To grasp the meaning of the Nice Treaty voting system – I replied – just set the population figures against votes allocated to EU states. Italy, Poland, and the Netherlands, have some 58, 38, and 16 million citizens, respectively. Italy has got 29 votes and the Netherlands 13. Now notice that the populations of Poland and Spain lie almost exactly in the middle between Italy and the Netherlands. Thus, one might expect that Poland and Spain would receive 21 votes each rather than 27 as in the Nice Treaty.
>
> *Why are we given that many?* The Big Four has invited Poland and Spain to the core of the enlarged EU to form the Big Six.
>
> *Are you sure that it's not a trick to make EU membership more attractive to us?* As it were, what we are promised is to share political influence with the Big Four.
>
> *Do you believe that the promise is going to be kept*? No doubt, the Nice Treaty is a provisional solution yet I think that there will be no push for changing it too soon.
>
> *Why*? As the enlargement is approaching, old an new Europe is more and more apprehensive of the consequences of the historic decision made by European political elites. Any attempt to undermine the Nice Treaty before it has come into effect might only produce more distrust within EU-25.

Such was the end of our discussion. I voted `Yes' in the accession referendum, she voted `No.'

What happened later in 2003 has shown that competence in game theory and political intuition are two different things. The Convention has ruled that the Nice game is to be replaced with a game under which largest countries gain too much power. Not only the Big Six has vanished but the Big Four has become Three Plus One. `Eurooptimism' shown before the referendum and later disappointment at the Constitution draft were typical of a host of educated and politically active Poles. Most of them, including myself, still support European integration despite being mocked at by the `europessimists'. Not only the latter but many `eurorealists' welcomed the words **Niza o muerte** uttered in the Polish parliament by one of the leaders of a party that is now the strongest rival of the ruling postcommunists. It wasn't a surprise that over 100 scholars and artists along with few legendary public figures addressed (November 17, 2003) Polish and European public opinion with an appeal to stand by the Nice Treaty.

When I traced among those who signed the address the names of few Polish scientists who deal with the theory of voting games, I thought that the hunch I had was correct. Poland and Spain would lose a lot if the Constitution Treaty were approved at the EU Brussels summit in December 2003. Fortunately, Poland and Spain did not yield to the pressure of Franco-German coalition, and the problem of how to define the European Council voting game remains open until today.

To join the discussion over the issue as an independent expert, I had to write a computer program for calculating power indices. My aim was also to prepare a **teaching aid for the game theory course** I have run for more than a decade at the Jagiellonian University's Institute of Sociology. Working on my program, I browsed in the Internet in search of data-based reports and theoretical papers on voting games. At the very beginning, I visited the game theory website at the University of Sevilla. The correspondence with Professor J.M. Bilbao helped me improve my program. As regards Poland, G. Lissowski, a fellow mathematical sociologist from the University of Warsaw, made available to me the preprint of an article that his collaborator M. Jasiński was going to publish in *Rzeczpospolita*. Meanwhile, I was working on my own article for Kraków's daily *Dziennik Polski*. In my paper which appeared January 28, 2004, I compared the values of the Banzhaf index for the Nice Treaty game and the EU Constitution game. I pointed to Germany's rise in power as the most important intended or unintended consequence of implementing the `population principle.'

I had not been fully aware of the gist of the EU reform until I found on Internet a paper presented in June 2001 by F. Bobay at French-German Economic Forum. Bobay quotes Jean Monnet's record of his meeting with Konrad Adenauer in April 1951.

> **Monnet**: *I am authorized to propose you that the relationship between Germany and France within the Community be governed by the* ***parity principle*** *in the Council, as well as in the Assembly and in all European institutions, current or future, whether France's participation includes the Union francaise and whether Germany be that of the West or reunified*.' […].
>
> **Adenauer**: *I am happy to give my full agreement to your proposal because I don't conceive the Community without total parity.*

I can understand that a politician will evaluate a given voting game, first of all, with respect to how large value a power index attains for his country. A game theory expert must pay more attention to the game's efficiency and power distribution as a whole. I won't help you if you ask me to tinker with weights or quota so as to satisfy just one country or a group of countries. What is needed is a generally acceptable voting system. To find a workable solution, the EU states must first agree on the guidelines. If the Big Four reject the parity principle, then the smaller states must respect this decision and seek a `parametric' solution. A country's population, as it were, is a fairly natural parameter on which to base weight allocation. But how to do this? An answer that is certainly worth attention was given by Słomczyński and Życzkowski at the end of January 2004. Their first article appeared in *Rzeczpospolita* two days after mine. Like me the authors were then absent in the Web, so I didn't know about their work. When I became familiar with the mathematical foundations of their proposal I gave my strong support to it in the **public debate** which took place March 29, 2004 in Kraków.

First, let me remark that the authors are not going to `die for Nice' as can be seen from their vote allocation displayed below.

	{D}	{F,GB,I}	{E,Pl}	{Nl}	Total
Nice Treaty votes	29	29	27	13	321
Square root votes	33	28	23	15	321

Even if Poland and Spain have moved toward the Netherlands, they are still nearer to the Big Four than to the `population' leader of 19 smaller countries. Notice also that the relative square root weights (33/321=.103 and 28/321=.087) do reflect population disparity within the Big Four but they come closer to each other than the respective relative linear population weights (82/452=.18, 60/452=.13). Look now at the values of the Banzhaf-Coleman index for the two games at issue and two compromise games.

		D	I	PI	NI	Efficiency
Nice	(13/241/620)	836	836	798	430	.020
P-62	(—/199/——)	1028	874	717	460	.169
55%+55%	(14/——/550)	941	693	547	378	.230
Const.	(13/——/600)	1331	929	677	359	.225

Italy and Poland represent here the groups {F,GB,I} and {E,Pl}. The games and index values are shown here in the form required by my program.

Notice that the Banzhaf-Coleman index values for the P-62 game are almost equal to relative weights; for instance (the case of Germany), 33/321=.1028. It is a consequence of Słomczyński and Życzkowski's choice to set the relative quota (qualified majority of votes) to the level at which relative weights coincide with the B-C index values. This special value of *q* was found to be equal to .62. Note that this quota entails a fairly high efficiency of the P-62 game (17%) so that the main deficiency of the Nice game is corrected.

To conclude, **I would strongly advocate the choice of P-62** as a system which allows to settle the dispute over political representation of the EU population structure. Mathematical elegance, simplicity, and transparency of the solution proposed by my colleagues should not go unnoticed, either.

Is there a chance that politicians will listen to the scientists and write a happy end to this story?

7. BIBLIOGRAPHY

I didn't have enough time to do a more systematic search of the vast literature on voting games or look through all materials available on Internet. The exposition of the theory of voting games given here in Sections 1-3 builds on the definitions found in Straffin's book *Game Theory and Strategy* (1993). In April 2004, I consulted Felsenthal and Machover's monograph *Measurement of Voting Power* (1998), earlier not available in Kraków. The authors' axioms 1-4 (p. 11) for a `proper simple voting game' are equivalent to axioms A1, A2, A3 given here, yet my terminology a little differs from theirs and from that most commonly used as I infer from the theoretical introduction given at the **Voting Power and Power Index Website** at the University of Turku (http://powerslave.val. utu.fi/). I suppose that my approach is not novel, so help me trace relevant sources so that I could provide in this Section a regular list of references and possibly re-edit the whole text to give it a publishable form.

Updated May 2004 Tadeusz Sozanski