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On Winning and Blocking Power in Voting Games

Abstract

A *voting game* (N,$\boldsymbol{W}$) is a set N of *voters* with a family $\boldsymbol{W}$ of its subsets such that: (1) $N\in\boldsymbol{W}$; (2) If $C\subset C'\subset N$ and $C\in\boldsymbol{W}$, then $C'\in\boldsymbol{W}$; (3) If $C\in\boldsymbol{W}$, then $N-C\notin\boldsymbol{W}$. The set $\mathcal{P}(N)$ of all subsets of N consists of 3 pairwise disjoint sets: $\boldsymbol{W}$, $\boldsymbol{L}=\{C: N-C\in\boldsymbol{W}\}$, and $\boldsymbol{B}=\mathcal{P}(N)-\boldsymbol{W}-\boldsymbol{L}$, with elements termed *winning*, *losing*, and *blocking coalitions*, respectively. A winning/blocking coalition C is called *minimal* if no proper subset of C is in $\boldsymbol{W}/\boldsymbol{B}$. In practice, voters are assigned positive *weights* p_i and $\boldsymbol{W}$ is defined – given a *quota* $q>\frac{1}{2}p(N)$ - as the set of Cs such that $p(C)\geq q$; $p(C)=\sum p_i$ over all $i\in C$. Double or triple majority voting systems (e.g. the ones designed for the European Council) are those defined as *intersections* of two or more *weighted voting games*; if (N,$\boldsymbol{W}_1$) and (N,$\boldsymbol{W}_2$) are voting games, then (N,$\boldsymbol{W}_1\cap\boldsymbol{W}_2$) is a voting game as well.

Measuring the degree to which group decisions depend on each voter is the key topic of the *theory of voting games*. Its exposition usually begins from the warning that *voting power* must not be confused with a voter's relative weight $p_i/p(N)$, but it should be construed as dependent on the number of winning coalitions in which voter *i*'s presence is necessary to remain winning. Next, one proceeds to define power indices of which two, the *Banzhaf* and *Shapley-Shubik*, are favored by most *academic* analysts.

The measurement of voting power became a hot issue in mathematical political science when the decision rules for the enlarged EU were set up by the Nice Treaty (2001). Since then various single or multiple majority voting systems have been proposed for the EU Council and analyzed by many experts (see http://www.cyf-kr.edu.pl/~ussozans/voting.htm). In June 2004, some 50 scientists advocated (in a letter to the governments) a weighted voting game with weights computed as square roots of the EU states populations. This meant the rejection of both the crude demographic weights, finally retained in the Constitution Treaty, and "political" weights reflecting a negotiated division of power. The scholars argued that the system based on Penrose's theorems more faithfully renders democratic principles and yields a flatter power distribution than the Constitution game.

The aim of this paper is not to convert politicians to scientific methods of constructing voting systems, but to propose a scientific reconstruction of their priorities and to explain such intriguing outcomes as the consent of France, UK and Italy to the game which – *if* voting power is calculated by means of classical indices – gives Germany a big power advantage over them. The author claims that what the negotiators wanted to maximize for their states was, in fact, blocking power described by the following statement: the *blocking power* of an actor decreases with increasing number of other actors needed to form with him a minimal blocking coalition and increases with increasing number of voters from among whom he may choose partners for *small size* minimal blocking coalitions. The *winning power* has a similar meaning.

The paper brings a mathematical elaboration of these two relatively independent facets of voting power and presents an analysis of the EU Constitution game in terms of certain new indices.

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http://www.cyf-kr.edu.pl/~ussozans/

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