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THE THEORY OF BLOCKING POWER AS THE KEY TO THE UNDERSTANDING OF THE HISTORY OF DESIGNING VOTING RULES FOR THE EU COUNCIL[1]

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ABSTRACT

The theory presented in this paper, unlike the classical theory of voting power, makes a distinction between blocking power and winning power and relates the blocking power of a voter to the number of small-size blocking coalitions the voter can form with other voters. The theory of blocking power sheds light on the practice of designing voting systems for EU Council of Ministers from the beginning to the recent dispute over the Nice and Lisbon voting systems and alternative systems based on square root weights. The paper also offers a method for analyzing blocking opportunities the members of an assembly have in their bilateral relations. The method is illustrated by the change in Franco-German balance of blocking power that would follow the replacement of Nice voting rules with the Lisbon system.

KEY WORDS ● voting game ● voting power ● blocking power ● EU Council

1. Introduction, or on politicians and mathematicians

Politicians construct voting systems by negotiating among themselves the rules of the games they are going to play. In doing so, they concern themselves with the distribution of *voting power*. When a member of a given decision-making assembly insists that voting rules be designed so as to maximize his own power, he must somehow define the quantity to be maximized. No politician can do this without 'theorizing', no matter whether he chooses to theorize by himself or seeks help from mathematicians or mathematical political scientists doing research on voting games.

Mathematicians study *structural* properties of mathematical objects they call *voting games.* In particular, they analyze voting games which are mathematical *models* of voting systems constructed by the politicians. Such a model is obtained by translating voting rules from the *legal* language into the *set-theoretic* formal language of mathematics. The example given below shows that the politicians like to construct voting systems with pretty complex structure which may be hard to understand for themselves without the help of experts.

[1] The theory expounded in Sections 3 and 4 of this paper was for the first time presented on the '2nd Polish Symposium on Econo- and Sociophysics', Kraków, April 21–22, 2006, and subsequently, with added Section 5, at the International Workshop 'Distribution of Power and Voting Procedures in the European Union', European Center Natolin, Warsaw, October 12–13, 2007. Some results appeared earlier in a less technical form in two papers (Sozański 2007a,b) in Polish.

The *legal definition* of the voting system for the EU council given in the Lisbon treaty Decision rules are stated so as to make them applicable under varying number of EU members.	A *mathematical model* of this voting system implemented for EU-27 Integer weights are used to enable exact calculations of the number of coalitions of particular types
Article 16 4. *As from* 1 *November* 2014*, qualified majority shall be defined as at least* 55% *of the members of the Council, comprising at least fifteen of them and representing Member States comprising at least* 65% *of the population of the Union.* *A blocking minority must include at least four Council members, failing which the qualified majority shall be deemed attained.*	**Voting game** $(H_1 \cap H_2) \cup H_3$ where H_1, H_2 and H_3 are three *weighted voting games* with the same set of voters {1,...,27}. H_1 is a weighted voting game with relative population weights which add up to 1000 and quota q_1=650. H_2 and H_3 are two 1 *voter*– 1 *vote* games with quotas q_2=15 and q_3=24.

The mathematicians have defined various coefficients of voting power with the intention to provide the politicians with measurement tools for practical use. Although inspiration for formal theorizing has to a greater or lesser degree been provided by political practice, the ‘scientific’ concept of voting power has often held little appeal for the politicians. Why? You will find an answer in this paper which I wrote to help the two ‘tribes’ speaking different languages better understand each other.

2. Three theories of voting power

2.1 *Naive theorizing*

The scope of this most popular way of theorizing is limited to voting systems defined by assigning *weights* (number of nominal votes, share of the total population, etc.) to *voters* and setting a *quota q*, or the threshold that must be attained by the total weight of a set of voters in order that the set become a *qualified majority* entitled to pass any bill.

Under the naïve approach, the *power* of a voter is equated with the voter’s weight or relative weight. Then, the power distribution does not depend on the choice of quota and the role of the latter reduces to determining the range of qualified majorities, which has to do with the *efficiency* of the voting system.

2.2 *Classical mathematical theory of voting power*

The classical theory of voting power began from seminal papers by Penrose (1946), Banzhaf (1965) and Coleman (1971). The theory's key concept is critical membership in a winning coalition.

Let $\mathcal{N}$ denote the set of *voters* (actors/players) and $\mathcal{W}$ the set of winning coalitions. To simplify notation, let $\mathcal{N}=\{1,\dots,n\}$. A subset C of $\mathcal{N}$ is a *winning coalition* if the support of all members of C suffices – by virtue of certain ‘voting rules’ – to pass any bill.

DEFINITION 1. Voter i is a *critical* (decisive/pivotal/swing) member of a winning coalition C ($C\in\mathcal{W}$) if i is a member of C ($i\in C$), and $C-\{i\}$ is not a winning coalition ($C-\{i\}\notin\mathcal{W}$).

That is, if a member i of a winning coalition C fails to vote for a bill, the votes of the remaining members of C will no longer suffice for passing it. Under the classical approach, the *voting power* of an actor i is directly proportional to the *number* $w_c(i)$ *of winning coalitions containing voter i as a critical member*.

The number $w(i)$ of *all* winning coalitions containing i can also be applied to measure voting power because it is related to $w_c(i)$ through the following formula discovered by Dubey and Shapley (1979)

$$w(i)=½(w_c(i)+w)$$

where $w=|\mathcal{W}|$ is the number of all winning coalitions (the number of elements of a finite set Z will be noted $|Z|$).

Proof. Consider the sets $\mathcal{W}(i)=\{C\in\mathcal{W}: i\in C\}$, $\mathcal{W}_c(i)= \{C\in\mathcal{W}(i): C-\{i\}\notin\mathcal{W}\}$, $\mathcal{W}^*(i)=\{C\in\mathcal{W}: i\notin C\}$, $\mathcal{W}_c^*(i)= \{C\in\mathcal{W}(i): C-\{i\}\in\mathcal{W}\}$. Since the assignment $C\rightarrow C\cup\{i\}$ is a 1–1 mapping of $\mathcal{W}^*(i)$ onto $\mathcal{W}_c^*(i)$, we get $w^*(i)=w_c^*(i)$, which equation together with $w=w(i)+w^*(i)$ and $w(i)=w_c(i)+w_c^*(i)$ yields the Dubey-Shapley formula.

The most widely used *normalized* (with values in the [0,1] interval) coefficient of voting power is the Banzhaf index $\beta(i)$. It is obtained by dividing $w_c(i)$ by the sum of $w_c(j)$ over $j=1,\ldots,n$. The Banzhaf index is a measure of *relative* power, which means that its values over the set of voters add up to 1.

2.3 *Mathematical theory of blocking power*

The third theory shares all basic concepts with the classical theory, yet its most fundamental term is *blocking coalition* ('blocking minority' in EU documents)

DEFINITION 2. C is a *blocking coalition* if (i) $N–C$ is not winning ($N–C\notin\mathcal{W}$) and (ii) C is not winning ($C\notin\mathcal{W}$), that is, neither non-members nor members of C form a winning coalition.

Condition (i) means that coalition C is given the power to prevent any bill from being passed. If all members of C refuse to vote for a bill, then it will not be passed, even if all remaining voters (members of $N–C$) vote for it. Since condition (i) is also satisfied by all winning coalitions (because C and $N–C$ cannot both be winning), condition (ii) must be added in order to distinguish between winning and blocking coalitions. Condition (ii) implies that if all members of C vote for a bill, but all other voters fail to vote for it, then the bill will not be passed. Blocking coalitions are less powerful than winning coalitions; the latter can both block initiatives of non-members and push through their own initiatives.

Our definition of a blocking coalition brings back to life the original meaning given to this term by Lloyd Shapley (1962). 'That sense – say Felsenthal and Machover (1998: 23) – agrees with common political parlance, in which the term is used to refer to a coalition that is able to stop a bill being passed but cannot force one through. However, subsequent usage in the voting-power literature has shifted to the broader sense of blocking, which we adopt here.' This 'broader sense' which actually prevails in the literature is obtained by defining a blocking coalition as any subset C of N such that $N–C$ is not winning. The theory of blocking power that I'm going to develop later in this paper breaks with this tradition and builds on the following three general heuristic principles:

- blocking power should be distinguished from winning power;
- blocking power should be measured with the use of blocking coalitions;
- blocking coalition should be defined in agreement with political practice.

Classical theory, which does not distinguish between two varieties of voting power, offers the ratio $w_c(i)/w$ as a measure of 'preventive power.' This coefficient, defined by James Coleman (1971), however based on counting winning coalitions, has in fact to do with blocking power because it assumes the maximum value of 1 for a voter *i* if and only if *i* is a *vetoer* (that is, by definition, $\{i\}$ is a blocking coalition) or a *dictator* (that is, $\{i\}\in\mathcal{W}$). Dictatorship and the right of veto are extreme cases of winning and blocking power, respectively. While there can be only one dictator, maximal blocking power can be granted to all members of an assembly, as is the case with the *consensus game* having only one winning coalition made up of all players.

2.4 *Abstract voting games*

In the interest of nonmathematical readers who usually abhor too abstract discourse, I have not so far explicitly distinguished between two terms: 'winning coalition' and 'qualified majority', the latter term being used by those who define decision rules in the language of law. However, not all historically known voting systems, including the one defined by point 4 of Article 16 of the Lisbon treaty, have been designed solely by assigning weights to voters and setting a quota. Therefore, for the sake of generality, I must introduce now the theory of voting games as an axiomatic mathematical theory.

An abstract *voting game* is a *mathematical object* of the form $(\mathcal{N},\mathcal{W})$, where $\mathcal{N}$ is a finite set of *voters*, and $\mathcal{W}$ is a collection of subsets of $\mathcal{N}$ called *winning coalitions*. The distinction between $\mathcal{N}$ and N in notation is to show the reader that the set which is referred to as the *assembly* $\mathcal{N}$ of voters (it's the *base set* of the *mathematical object* $(\mathcal{N},\mathcal{W})$ with *structure* $\mathcal{W}$) is also a subset of N of $\mathcal{N}$, being then referred to as the *grand coalition*.

The starting point for building the *mathematical theory of voting games* are not concrete *voting rules*, but abstract *axioms* assumed to be met by $\mathcal{W}$. The following axioms seem most convenient insofar as one would like to develop a formal theory that would serve as a basis for typical political applications:

A_1 $\mathcal{W}\neq\varnothing$ (there exists at least one winning coalition);

A_2 If $C\in\mathcal{W}$ and $C\subset D\subset \mathbf{N}$, then $D\in\mathcal{W}$ (any set of players containing a winning coalition is also a winning coalition);

A_3 If $C\in\mathcal{W}$, then $N-C\notin\mathcal{W}$ (the non-members of a winning coalition do not form a winning coalition.

The mathematicians love general concepts and general theorems. Thus, even those willing to attract nonmathematical readers (Felsenthal and Machover 1998, Straffin 1993) begin theory building from defining a *simple voting game* as a mathematical object which meets only two axioms A_1 and A_2 (A_1 is usually replaced by B_1: $N\in\mathcal{W}$, which under our axiomatics follows from A_1 and A_2). Axioms A_3 is used then to define a particular class of simple voting games, referred to as *proper* simple voting games. In this paper, for convenience, I use the term *voting game* for this special case corresponding to a widely accepted *political decision rule* that two *contradictory* bills, the one supported by C and the other supported by $N–C$, may not be passed simultaneously, which could be possible if Axiom A_3 did not hold.

A coalition will be called *losing* if its complement $N–C$ is winning. Note that in classical theory, the term 'losing' is used synonymously with 'not winning'. The axioms imply that C is losing if and only if $N–C$ is winning. Hence $|\mathcal{W}|=|\mathcal{L}|$ where $\mathcal{L}$ stands for the set of losing coalitions. The remaining subsets of $\mathcal{N}$ are blocking coalitions. Let $\mathcal{B}$ denote their set. We have $2w+b=2^n$ where $b=|\mathcal{B}|$.

3. The measurement of blocking power

3.1 *Can blocking power be measured by analogy with winning power*?

Once $w(i)$ is a measure of winning power, can $b(i)$ – the number of blocking coalitions containing actor i – be used to construct an index of blocking power? The answer is negative due to the following formula

$$b(i)=2^{n-1} - w$$

Proof. Notice that the mapping $C\to N–C$ of the set of all coalitions onto itself establishes a one-to-one correspondence between $\mathcal{B}(i)=\{C\in\mathcal{B}: i\in C\}$, $\mathcal{B}^*(i)=\{C\in\mathcal{B}: i\notin C\}$, two disjoint sets which make up $\mathcal{B}$. Therefore, $b(i)=½b$, but $b=2^n–2w$.

Consider in turn the parameter $b_c(i)=|\mathcal{B}_c(i)|$ where $\mathcal{B}_c(i)=\{C\in\mathcal{B}(i): C–\{i\}\notin\mathcal{B}\}$ is the set of blocking coalitions containing voter i as a critical member, where 'critical' means that if actor i leaves a blocking coalition C, then $C–\{i\}$ is not a blocking coalition (then it must be losing). We prove the following fact about $b_c(i)$:

$$b_c(i)\leq w_c(i),\ b_c(i)=w_c(i) \text{ if and only if, for any } C\in\mathcal{W}_c(i),\ C–\{i\}\in\mathcal{B}$$

Proof. Clearly, the mapping $C\to(N–C)\cup\{i\}$ assigns different sets to different sets, hence $b_c(i)\leq w_c(i)$ provided that $C\in\mathcal{B}_c(i)$ implies that $(N–C)\cup\{i\}\in\mathcal{W}_c(i)$. If $C\in\mathcal{B}_c(i)$, then $C–\{i\}\notin\mathcal{B}$ and consequently $C–\{i\}\in\mathcal{L}$, which implies that $N–(C–\{i\})=(N–C)\cup\{i\}\in\mathcal{W}$, but $N–C\in\mathcal{B}$ because $C\in\mathcal{B}$, so that $(N–C)\cup\{i\}\in\mathcal{W}_c(i)$.

The above iff condition, which means that the defection of a coalition member can never result in a direct transition from winning to losing coalition, holds true for most known *weighted voting games*, or voting games obtained by assigning to any voters positive numbers $p_1,\ldots,p_n$, called *weights*, setting a number q (*quota*) such that $½(p_1+\ldots+p_n)< q \leq p_1+\ldots+p_n$, and defining $\mathcal{W}$ as the set of all subsets C of $\mathcal{N}$ such that the sum of weights over C equals at least q. As a consequence, my first idea to define with the use of $b_c(i)$ the *Banzhaf-like index of blocking power* has turned out of little practical value, as blocking power would almost always be equal to winning power.

3.2 *Minimal winning and minimal blocking coalitions*

DEFINITION 3. A winning (blocking) coalition C is called *minimal* if no proper subset of C is winning (blocking). Formally, $C\in\mathcal{W}$ $(C\in\mathcal{B})$ is minimal if for any D such that $D\subset C$ and $D\neq C$, $D\notin\mathcal{W}$ $(D\notin\mathcal{B})$.

Equivalently, C is minimal if every member of C is critical, that is, C has no redundant members whose defection would not change the coalition type. We define in turn *structural parameters* based on counting minimal blocking coalitions.

b_m – number of all minimal blocking coalitions
$b_{m,k}$ – number of minimal blocking coalitions of size k
$b_m(i)$ – number of minimal blocking coalitions containing voter i
$b_{m,k}(i)$ – number of minimal blocking coalitions of size k containing voter i

If blocking power were to be measured by the number $b_m(i)$ of all minimal blocking coalition containing player i, one would obtain the following apparently strange result for the weighted voting game used by the Council of Ministers in EU-15 (q=62).

Table 1. Minimal blocking coalitions in EU-15

p_i (weight)	$b_m(i)$
$p_1=p_2=p_3=p_4=10$	324
$p_5=8$	334
$p_6=p_7=p_8=p_9=5$	489
$p_{10}=p_{11}=4$	494
$p_{12}=p_{13}=p_{14}=3$	485
$p_{15}=2$	375

Small countries surpass large countries in the number of minimal blocking coalitions because they can form many such coalitions among themselves. However, since these coalitions must have many members, their formation may turn out more difficult than the formation of smaller size coalitions in which strong players ally with weaker players.

3.3 *Measuring blocking power with the use of small minimal blocking coalitions*

The above reasoning leads to the idea that the blocking power of a voter should depend on the smallest size of a minimal blocking coalition the voter can form with other voters and on how many alternative *small* minimal blocking coalitions are available to him.

What size of a minimal blocking coalition, besides $k_{min} = \min\{k: b_{m,k}>0\}$, should be considered small? Such a question should be asked the users of a given voting system. Every player i would probably agree that any minimal blocking coalition of the size $k_{min}(i)= \min\{k: b_{m,k}(i)>0\}$ is small, as *for him* it is the smallest possible size.

If $b_m(i)$=0, or $b_{m,k}(i)$ for any k, we put $k_{min}(i)$=0. If a voter i is a *dummy* (the case of Luxembourg in EU-6 being the most famous example), that is, if $w_m(i)$=0 ($w_m(i)$ is the number of minimal winning coalitions containing i), then $b_m(i)$=0, which means that a player deprived of any winning power, doesn't have blocking power, either. For now I can't prove or disprove the converse implication, or that of the form if $b_m(i)$=0, then $w_m(i)$=0, which, of course, may hold true only for those games in which $b=|\mathcal{B}|>0$ (if this implication were true, then the case of the smallest blocking power and that of smallest winning power would coincide). There exist voting games in which b=0, or $\mathcal{B}=\varnothing$, that is, the players have no opportunity for blocking. These games, called *strong*, are most efficient, *efficiency* (named by Coleman the 'power of a collectivity to act') being defined as the ratio of w to $2w+b$.

We define *small* minimal blocking coalitions as those of the size ranging from $k_{\min}$ to the maximum, noted $k_{\max}$, of $k_{\min}(i)$ over all *i*.

Let us define in turn the simplest coefficient of blocking power as the *ratio of the number of small minimal blocking coalitions containing voter i to the number of all small minimal blocking coalitions*, symbolically

$$\gamma(i) = \frac{\sum_{k=k_{\min}}^{k_{\max}} b_{m,k}(i)}{\sum_{k=k_{\min}}^{k_{\max}} b_{m,k}}$$

This parameter, which disregards the size of blocking coalitions, could be refined by means of the same method as the one which led to defining the Deegan-Packel coefficient of winning power (Deegan and Packel 1979). However, the latter coefficient is based on counting *all* minimal winning coalitions, even if they are weighted by size.

4. The blocking structure of a voting game

4.1 *Formal properties of the blocking structure*

The constructors of voting systems for the EU Council seem to be little interested in *methods of quantifying* blocking power. They are more concerned with the *shape of blocking structure* with focus on its *lowest level*, or the distribution of the number of minimal blocking coalitions of the smallest size $k_{\min}$. The set of such coalitions of which the number is usually pretty small can often be determined by political users without the help of experts.

DEFINITION 4. For any voting game with *n* players, the *blocking structure* is the sequence of sequences $(b_{m,k}(i)$: $i=1,\dots,n)$, $k=k_{\min},\dots,k_{\max}$.

In describing the shape of a blocking structure, one needs to take into account the following formal properties or parameters:

- the smallest size of a minimal blocking coalition; this parameter has always been considered important in designing voting games for the EU Council ($k_{\min}$ was always equal to 2, 3 or 4 with the tendency to be raised with successive EU enlargements);
- the number of levels ($k_{\max}-k_{\min}+1$) in the blocking structure; in EU games it has never exceeded 3;
- the number of voters with $b_{m,k}(i)>0$ at level *k*; the set of players who take part in minimal blocking coalitions of the smallest size will be referred to as the *premier league*;
- even vs. uneven distribution of non-zero $b_{m,k}(i)$ on each level;
- last but not least, *regularity* (to be defined below) or irregularity of the blocking structure.

4.2 *Regularity of the blocking structure*

DEFINITION 5. A *weighted voting game* is said to have a *regular blocking structure* if the order of voters 1,...,*n* with respect to the weights $p_1,\dots,p_n$ is *consistent* with their order with respect to $b_{m,k}(1),\dots,b_{m,k}(n)$, for any *k* from $k_{\min}$ to $k_{\max}$.

Consistency is defined as the absence of pairs of distinct players *i* and *j* such that $p_i>p_j$ and $b_{m,k}(i)<b_{m,k}(i)$. Thus, if actor *i* is more powerful than actor *j* according the naive theory of voting power, then *j* will not move ahead of *i* in the ordering of players with respect to the number of minimal blocking coalitions of size *k*. As a consequence, if $p_i>p_j$, then $b_{m,k}(i)>b_{m,k}(j)$ or $b_{m,k}(i)=b_{m,k}(j)$.

The definition of regularity can be extended to any voting game constructed as an *intersection of two or more weighted voting games with consistent sequences of weights.* The *intersection* of two voting games $(\mathcal{N},\mathcal{W}_1)$ and $(\mathcal{N},\mathcal{W}_2)$ over the same assembly $\mathcal{N}$ of voters is defined as a voting game of the form $(\mathcal{N},\mathcal{W}_1\cap\mathcal{W}_2)$ where $\mathcal{W}_1\cap\mathcal{W}_2$ is the intersection of the sets $\mathcal{W}_1$ and $\mathcal{W}_2$ of winning coalitions of the two games.

Lastly, for any *abstract* voting game with blocking structure having at least two levels, we define regularity by the condition that, for any distinct k_1 and k_2 such that $k_{min}\leq k_1,k_2\leq k_{max}$, the orders of the set of players $\{1,...,n\}$ with respect to $b_{m,k1}(i)$ and $b_{m,k2}(i)$ are consistent with each other.

If a two-level blocking structure of a voting system is not regular, then a voter who occupies a high position on one level and low position on the other level may have difficulty with locating his place in the overall power hierarchy of blocking power as well as with defining this hierarchy.

4.3 *The blocking structure of the Nice voting game*

To show an example of the troubles produced by irregularity I analyze below the Nice 'triple majority voting system' that is currently in use. Its mathematical model is the voting game of the form

$$G = G_1\cap G_2\cap G_3$$

Where

- G_1 is a weighted voting game obtained by distributing 345 'nominal votes' among the members of EU-27 and setting the quota to 255;
- G_2 is 1 voter–1 vote weighted voting game with quota 14;
- G_3 is a weighted voting game which corresponds to the third voting system defined in the Nice treaty by the following statement: *When a decision is to be adopted by the Council by a qualified majority, a member of the Council may request verification that the Member States constituting the qualified majority represent at least 62% of the total population of the Union. If that condition is not shown to have been met, the decision in question shall not be adopted.*

The value of the Banzhaf index does not depend too much on whether we use in calculations absolute or relative population weights. The study of blocking structures, which is aimed at determining exact numbers of small minimal blocking coalitions, requires that integer weights and quota be used as input to the computer programs. The program (POWERIND) I wrote transforms the population data into integer weights which add up to 1000. Then 620 is the integer counterpart of the relative quota of 62% given in the above clause quoted from the Nice treaty. Throughout this paper we use the Eurostat population data which were official input to decision procedures during the German presidency in the first half of 2007.

Formally, the Nice game *G* has a 3-level blocking structure, but the players except the weakest one (Malta) probably don't take into account the third level of minimal blocking fives in estimating the size of their blocking power.

Table 2. The blocking structure of the Nice game

EU–27	Nice wght	Population		$bm_k(i)$		
		in 1000	wght	k=3	k=4	k=5
1. Germany	29	82438	167	4	90	651
2. France	29	62886	128	3	109	663
3. UK	29	60393	122	2	128	666
4. Italy	29	58752	119	2	125	678
5. Spain	27	43758	89	1	124	563
6. Poland	27	38157	77	0	136	590
7. Romania	14	21610	44	0	16	678
8. Netherlands	13	16334	33	0	16	528
9. Greece	12	11125	23	0	16	405
10. Portugal	12	10570	21	0	16	405
11. Belgium	12	10511	21	0	16	405
12. Czech R.	12	10251	21	0	16	405
13. Hungary	12	10077	20	0	16	405
14. Sweden	10	9048	18	0	16	239
15. Austria	10	8266	17	0	16	239
16. Bulgaria	10	7719	16	0	16	239
17. Denmark	7	5427	11	0	12	76
18. Slovakia	7	5389	11	0	12	76
19. Finland	7	5256	11	0	12	76
20. Ireland	7	4209	8	0	12	76
21. Lithuania	7	3403	7	0	12	76
22. Latvia	4	2295	5	0	2	86
23. Slovenia	4	2003	4	0	2	86
24. Estonia	4	1345	3	0	2	86
25. Cyprus	4	766	2	0	1	88
26. Luxembourg	4	460	1	0	1	88
27. Malta	3	404	1	0	0	72
	345	492852	1000	4	235	1729

The shape of the blocking structure of G is jointly determined by G_1 and G_3. The 1 voter–1 vote game G_2 has no effect on the set of small minimal blocking coalitions of G.

For now, I don't know to what degree the set of *minimal blocking fives* depends on G_1 and G_3. However, one can easily prove that: (1) every blocking five must contain at least 2 out of 6 largest states; (2) every 2 members of the Big Six can block any initiative of 4 remaining members with the help of 3 weaker players. The examination of the set of minimal blocking threes and fours has led to the discovery of the following facts.

G has 4 blocking threes, all inherited from G_3 (the total integer population weight of each coalition is given in brackets): {Germany, France, UK}(417), {Germany, France, Italy}(414), {Germany, UK, Italy}(408), {Germany, France, Spain}(384).

All blocking threes contain Germany. Notice also that the total weight of {France, UK, Italy}, or the strongest three without Germany, equals 369, which reveals the political meaning of setting the quota in G_3 to 620 rather than 630 or 640.

The set of *minimal blocking fours* consists of 235 coalitions of which only 3 owe the property of blocking *solely* to the population game G_3. These 3 blocking fours are obtained by appending to the 3-state coalition {Germany, UK, Spain} one of 3 very small states, Latvia, Slovenia or Estonia. The total weights of the 4-player coalitions are 383, 382 and 381, respectively, so that their future is uncertain.

The remaining 232 minimal blocking fours in the Nice game come (not always exclusively) from game G_2 defined with the use of fixed political weights. Thus, the hybrid blocking structure of the Nice game will remain *stable* at least at the second level.

While Germany is the leader on the first level of blocking threes, it drops to the last position within the Big Six on the level of minimal blocking fours where Poland has unexpectedly taken the lead. On the level of minimal blocking fives, Poland and Spain do rather poorly, Italy and Romania are now ahead of all other players, which makes the blocking structure of the Nice game highly *irregular*.

5. Relational analysis of blocking power

5.1 *Four facets of the relationship between two members of a voting assembly*

Along with the *distributive* understanding of *power*, political scientists have always construed power as a *relational* concept, as illustrated by Dahl's definition (1957: 202–203): '*A* has power over *B* to the extent that he can get *B* to do something that *B* would not otherwise do.' I will show now how to formalize this idea in the context of the theory of blocking power in voting games.

Given the set S of all *small* minimal blocking coalitions in a voting game, we define four coefficients that are to depict different aspects of institutionally-based political relationship of two actors *A* and *B*. Let $S_{A\vee B}$ denote the set coalitions in *S* containing player *A or* player *B*, and $S_{A\wedge B}$ – the set of coalitions in *S* containing both *A and B*.

The ratio $|S_{A\vee B}|/|S|$ measures *structural importance* of the pair $\{A,B\}$ within the voting system. The ratio $|S_{A\wedge B}|/|S_{A\vee B}|$ is a measure of *system-forced potential cooperation* of *A* and *B* in blocking initiatives of other players. Both coefficients are symmetric, that is, $I_{AB}=I_{\mathrm{BA}}$ and $C_{AB}=C_{BA}$.

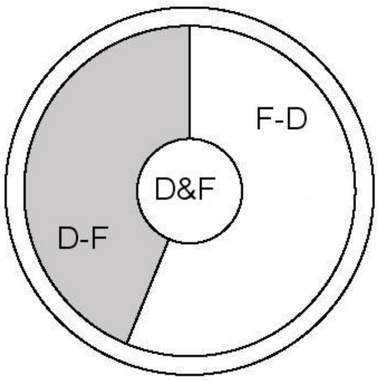
Let S_{A-B} stand for the set of coalitions in *S* containing *A* but *not B*, or those coalitions *A* can use to block *B*'s initiatives. S_{B-A} is defined similarly. The ratios $P_{AB}=|S_{A-B}|/|S_{A\vee B}|$ and $P_{BA}=|S_{B-A}|/|S_{A\vee B}|$ are measures of blocking power *A* and *B* have in relation to each other.

If $P_{AB}>P_{BA}$, *A* is said to have *blocking power advantage* over *B*. Notice that a player *i* has blocking power advantage over player *j* if and only if $\gamma(i)>\gamma(j)$, that is, the order of players with respect to the values of the blocking power parameter determines their unequal opportunities to block each other's initiatives. Thus, distributive and relational blocking power are related to each other.

5.2 *A politically interesting example*

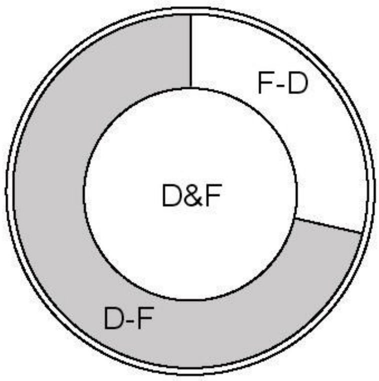
Chancellor Merkel and President Sarkozy might be interested to learn how the political relationship their countries (*D* and *F*) would change if the voting system defined in the Lisbon treaty were to replace the Nice voting system right now.

For the Nice game, let us take as *S* the set of minimal blocking threes and fours, for the Lisbon game, the set of minimal blocking fours.



The Nice Game

I_{DF} = 78.7
C_{DF} = 9.6
P_{DF} = 40.4
P_{FD} = 50.0



The Lisbon Game

I_{DF} = 96.9
C_{DF} = 36.0
P_{DF} = 46.4
P_{FD} = 17.6

What would be the consequences of implementing the Lisbon system? The values of our four indices (given above in %) dictate the following answer:

- Already high importance of the Franco-German contribution to the total EU blocking structure would increase even more;
- the potential for system-forced cooperation between these two members of the EU 'core' would considerably increase;
- but the current moderate power advantage of France over Germany would be replaced by big power advantage of Germany over France.

So far the Franco-German balance of power has been the cornerstone of the EU. Will the Union survive so radical change in this matter? Let the politicians answer the question posed by a mathematician.

6. How are the three power theories related to one another?

6.1. *Naive vs. classical mathematical theory*

Let p_i and p_j denote the weights of players i and j in a weighted voting game with quota q. If $p_i > p_j$, then $w(i) \geq w(j)$, which implies in turn that $w_c(i) \geq w_c(j)$ and $\beta(i) \geq \beta(j)$. Thus, the order of players with respect to weight and their order with respect to the value of the Banzhaf index are always consistent with each other.

Słomczyński and Życzkowski (2006) suggest to calculate a quota from the weights in such a way that the *relative weight* of each voter and the respective value of the Banzhaf index are approximately equal. They found a formula for such a quota for the case of weighted voting games with square root weights. For the square root game they designed for EU-27 the relative quota with this property equals 61.6%. If square root weights are represented in the form of integers which add up to 345, the absolute quota corresponding to .616 equals 213.

The use of such a quota may help political users reconcile their naive approach to voting power with the classical approach. However, the success of this strategy of promoting the mainstream mathematical voting power theory depends on whether the politicians agree to apply this theory in practice. A scholar can do nothing to gain acceptance for his approach when he or she hears from a politician: I'm sorry, but your way of measuring voting power much differs from mine.

The classical theory is not doomed to remain 'academic.' According to Dubey and Shapley (1979, p. 100): 'The main ideas underlying the game-theoretic approach to power eventually found wide legal acceptance; indeed, in New York State today, some of the county supervisorial boards are constituted according to a form of Banzhaf's index, in an attempt to equalize the representation of citizens living in municipalities of different size.'

If a choice between two or more games has to be made by political actors who will be the future players of the chosen game, then an actor's evaluation of each game in terms of how strong is his position in it depends on the power measure he uses. If a player's 'strength' is quantified by two different measures, then the consistency of the orderings generated by them does not suffice to find a *compromise solution* of the problem of which game to choose. The example given below provides a plausible explanation of why Germany refused to accept the *Jagiellonian game*, or the game with square root weights and the relative quota 61.6%.

Let us compare this game with two other voting games for EU-27, so differently evaluated by Poland and Germany, the Nice game and the Lisbon game. Under the naive approach the Nice 'triple majority system' is usually identified with its main component, or the game with political weights (nominal votes) and quota 255.

Table 3. The voting power of Germany in three games according to the naive theory and classical theory

Power measure \ Game	Nice	Jagiellonian	Lisbon
Integer weight	29	33	58
Relative weight	.0841	.0957	.1673
Banzhaf index	.0778	.0955	.1164

If you compute the mean of the Banzhaf index values for the Nice game and the Lisbon game, you will get .0971, which exceeds by .014 the respective value for the Jagiellonian game. Thus, the latter game can in fact be regarded as a compromise solution. But if you rely on the naive approach as probably did the German government and take into account relative weight or absolute integer weight (calculated so as to imitate Nice weights which add up to 345), you will arrive at a completely different conclusion: the 'Jagiellonian compromise' is by no means in the middle between two games considered best and worst for Germany. The values of the *Shapley-Shubik index* (the second most popular classical power coefficient, a special case of *Shapley value*), which in the three games are equal for Germany to .0874, .1001, .1592, also show that the Jagiellonian game is closer to the Nice game than to the Lisbon game.

6.2 *The naive way of measuring blocking power*

The naive theory which equates voting power with relative weight deserves its name 'naive' – I must say as a *mathematician*. As a *social scientist* – I would not reject this approach altogether, even if it was dismissed by Felsenthal and Machover (1998: 156) as 'widespread fallacy' to which 'even experts on voting power are not immune.' The way in which the players *themselves* calculate how strong they are may affect the results of the game. It may well be that the members of a voting body who for their weights are assigned high *political status* can easier find partners for winning coalitions and thus enjoy a greater *political power*, or have a greater *real* influence on collective decisions.

Politicians and their advisors – experts in constitutional law or non-mathematical political science– hardly ever go beyond naive theorizing. Since they have always been preoccupied with maximizing blocking power, they must have invented their own measure computed from the weights and quota. The naive coefficient of blocking power, known as the *share of blocking minority*, is defined as the ratio of a voter i's *weight* p_i to the *blocking threshold*. I learned about its common use by EU politicians from newspaper reports and Moberg's paper (2007).

To explain what is blocking threshold, notice that in a voting game with weights $p_1,...,p_n$ and quota q, the type of any coalition C can be easily determined by finding out which of three successive intervals contains the *total weight* $p(C)$ of C, or the sum of p_i over all members of C. The intervals which correspond to losing, blocking, and winning coalitions have the form:

$$[0, p(N)–q], \ (p(N)–q, q), \ [q, p(N)].$$

The lower bound of the middle interval, or $p(N)–q$ is usually referred to as the blocking threshold. When the weights and quota are integers, it is more convenient to define this quantity by means of the formula $r=p(N)–q+1$, which implies that any coalition C is blocking if and only if $p(C)\geq r$ and $p(C)<q$.

For example, for the weighted voting game used in EU-15, we have $p(N)=87$, $q=62$, so that $r=87–62+1=26$. Hence, for 4 strongest members of EU-15 (each of them had 10 nominal votes in the Council), the share of blocking minority equals 10/26=38.5%. The dissatisfaction of the Big Four with the outcome of negotiations in Nice might have had to do with the fact that their share of blocking minority, computed for the game with political weights (G_1) dropped to the value 29/91=31.9%.The game with population weights (G_3) was added to compensate for this loss, but only Germany benefited from making voting rules more complicated (167/381=43.8%; for France we have: 128/381=33.6%). For Spain, the shares of blocking minority, computed for the game used by the 'old Union' and for the political component of the game designed for the enlarged Union marginally differ (8/26=30.8%, 27/91=29.7%), both being much greater than the value (89/381=23.4) for the population component of the Nice game.

How to estimate Spain's total blocking power in the game that was defined as the intersection of three weighted voting games? Did this country get more power than it had in the Fifteen? The naive theory of winning and blocking power is unable to answer such questions. In addition, it yields odd results even for some voting games to which it can be applied. Compare two historical weighted voting games, the one used by the original Six and the other designed for the Nine which replaced the Six. In the first game, the value of the naive coefficient of blocking power for Luxembourg was equal to 1/6=16.7% despite the fact that this player did not belong to any minimal blocking coalition. By contrast, in the second game, Luxembourg is a member of 4 minimal blocking coalitions of size 4, but its share of blocking minority equals 2/18=11.1%.

6.3. *The classical theory and the theory of blocking power*

The practice of constructing voting systems by the politicians and their anonymous experts has always appeared to academic specialists devoid of any theoretical foundations. Such an opinion seems to be exaggerated, as amateurish political engineering has in fact gone beyond the limits of the naive approach I've just described. But it is true that insights and preconceptions behind tinkering with the blocking threshold and counting small size minimal blocking coalitions have so far remained without an adequate formalization. The aim of my paper is to make a significant step in this direction.

Analysts attached to the classical approach find it astonishing that the negotiators at EU summits are ready to argue till dawn about raising the quota by few points or appending an odd-looking clause to the treaty. Does it make any sense – they ask – to quarrel, once such minor modifications of the rules of the game may only negligibly affect values of the scientific measures of voting power. Indeed, let us compare consider the double majority game $H_1 \cap H_2$ (H_1 is the population game with quota 650 and H_2 is the 1 person – 1 vote game with quota 15) with the Lisbon game which has the form $H=(H_1 \cap H_2) \cup H_3$ (H_3 is the 1 voter–1 vote game with quota 24). If you compute the Banzhaf index for these two games of which the second differs from the first only with the *ban on blocking in threes* (this rule is formalized by defining H as the *union* of $H_1 \cap H_2$ and H_3), you will find out that the difference in power will not exceed .0001 for any player. But, as I'm going to show later in this paper, the condition that a *blocking minority must include at least four Council members*, has a dramatic effect on the blocking structure.

I have long wondered why the constructors of EU voting games have shown so little interest in the classical theory of voting power. Now I know that weak reception of this theory does not result from its mathematical sophistication. Does the concept of a critical player appear to laymen more obscure than that of a blocking minority? Certainly, not. I agree with Moberg (2007) that classical indices proved of little practical value because their calculation is based on the assumption that, in estimating a player's *winning* power, *millions* of theoretically possible and *equally probable* winning coalitions containing him as a critical member must be taken into account. Actually, what the players want to maximize is not winning but *blocking* power, and what really matters for a player is availability of alternative partners to form *small* minimal blocking coalitions. Their number is counted in hundreds rather than millions. Many small 'blocking minorities', unlike numerous 'anonymous' winning coalitions, can be quite concretely identified by the players for the sake of their political rather than mathematical calculations. What the politicians would like to know is how to find allies for blocking the initiatives of their rivals or whose support to seek to prevent blocking their own proposals.

The classical theory of winning power (wrongly equated with voting power *tout court*) and the theory of blocking power are, in fact, two branches of one axiomatic mathematical theory. The 'technologies' they generate are complementary to each other, yet in some cases they may prompt different practical solutions, first of all, as to the quota selection.

The next section offers a brief history of constructing voting games for the Council of Minister of the EU. It's going to be a *verstehende Geschichte*, as Max Weber would say. I will try to decipher the intentions of the constructors by analyzing the 'architecture' of the 'cathedrals' they built. Their activities become more understandable when seen in light of the theory of blocking power. The descriptions of voting systems and population data used in the analyses given in Section 7 come from Chapter 5 of Felsenthal and Machover's book (1998).

7. A history of voting systems designed for the EU Council of Ministers

7.1 *From the Six to the Twelve*

7.1.1 *How the story began some fifty years ago.* The germ of today's European Union consisted of 3 large states, 2 much smaller states, and 1 tiny state whose citizens then formed some .002% of the total population of the Six. Luxembourg got 1 nominal vote in the Council of Ministers, the Netherlands and Belgium – 2 votes, 3 largest states – 4 votes. The population of France (then the smallest country among the top Three) was almost exactly 4 times greater than that of the Netherlands, but the proportion of weights assigned to the two states was not 4 but 2, or the *square root* of 4. The largest proportion of populations within the Three was equal to 1.1. Although this could have been the sufficient reason to give the same weight to the top players, the decision of the father founders of the European Community was *political* par excellence. The *parity principle* was also applied to the second group: Belgium and the Netherlands received the same number of nominal votes.

With such an allocation of weights, the lowest winning threshold which allows the Big Three to outvote the Benelux equals 11. However, with such a quota the three weaker actors would be *structurally interchangeable*, and thus equally powerful. Indeed, if the Netherlands or Belgium is replaced with Luxembourg in any winning coalition, a winning coalition is obtained again. The constructors of the first voting game for the Council of Ministers could easily discover this fact by examining the set of all coalitions, which was not a too difficult task, since their number equals only $2^6=64$. Politicians turn to the naive theory of voting power when the number of coalitions is so great that one has to call a mathematician for help.

The quota was finally set at 12 votes. Belgium and the Netherlands gained power advantage over Luxembourg at the cost of depriving the smallest member of the Six of any winning or blocking power. Is it possible to define a voting game with 6 players so that the power ladder has three steps and the lowest one is not a redundant element in the whole decision mechanism? The problem can only be solved (I don't know the solution) by the inspection of all non-isomorphic voting games with 6 players. Two voting games $(\mathcal{N},\mathcal{W}_1)$ and $(\mathcal{N},\mathcal{W}_2)$ are *isomorphic* through a 1–1 mapping α of $\mathcal{N}$ onto $\mathcal{N}$ if for any $C\subset\mathcal{N}$, $C\in\mathcal{W}_1$ if and only if $\alpha(C)\in\mathcal{W}_2$.

There exist only 2 structurally distinct voting games with 2 players, the *consensus* game and the *dictatorial* game in which one of two players forms a unique minimal winning coalition. For n=3, there are 5 nonisomorphic games, two special games named above, and three other elementary games, the *duumvirate* game (two players form the only minimal winning coalition), the *hegemony* game (there are two minimal winning pairs having one voter in common; the latter has the right of veto, but needs cooperation of one of two other players to form a winning coalition), and the *majority* game (all three pairs are minimal winning coalitions).

7.1.2 *The first two enlargements.* Thus, Luxembourg could only persuade other states to vote according to its preferences. How it voted had no influence on the outcome of any voting. This deficiency was corrected when Great Britain, Ireland and Denmark joined the Six. In a new allocation of nominal votes, which was introduced for the Nine, old proportions of weights were preserved between two upper groups. To mark the difference between the second group and the third group containing two smaller countries, Denmark and Ireland received 3 votes. Luxembourg, with its outlying population size, formed now the fourth, one-element group. The four largest states were given 10 votes each, which value was both large enough to allow for more levels in the power hierarchy and convenient for calculating the ratios of weights.

Assume now that any winning coalition should consist of at least 5 states, that is, it should be at the same time a winning coalition in the simple majority 1 state–1 vote game. The minimum quota under which this condition is met equals 41. It is the value which was actually used by the constructors of the game for the Nine.

Table 4. Blocking structure in the games for the Nine, Ten, and Twelve

n	$p_1 \ldots p_n$	$p(N)$	q	r	$b_{m,k}(i)$		
					k=2	k=3	k=4
9	$p_1=p_2=p_3=p_4=10$ $p_5=p_6=5$ $p_7=p_8=3$ $p_9=2$	58	41	18	3 0 0 0	5 12 8 0	1 0 4 4
10	$p_1=p_2=p_3=p_4=10$ $p_5=p_6=p_7=5$ $p_8=p_9=3$ $p_{10}=2$	63	45	19	3 0 0 0	3 8 0 0	9 12 24 24
12	$p_1=p_2=p_3=p_4=10$ $p_5=8$ $p_6=p_7=p_8=p_9=5$ $p_{10}=p_{11}=3$ $p_{12}=2$	76	54	23		28 22 10 6 0	19 16 39 32 8

When Greece became 10th member of the Community, it was placed in the second group and received the same number of votes as Belgium and the Netherlands. The quota 45 was used instead of 41, or the minimum threshold guaranteeing that any winning coalition must have ½n members. Why? With q=41, the blocking threshold equals 63–41+1=23, which implies that any minimal blocking coalition must have at least 3 members. In the Six and the Nine, the minimum size of a blocking coalition was 2. In addition, in the Nine, the right to block in pairs was reserved for 4 top players. Under the weights used in the Nine and the Ten, the lowest blocking threshold with such a consequence equals 16 (if r were smaller, then any pair of players with weights 10 and 5 would be a blocking coalition). On the other hand, the highest blocking threshold which still enables blocking in pairs equals 20. Thus, one had to choose as quota for the Ten a number from the range from q=44 (r=20) to q=48 (r=16). Since quotas 44 and 45 generate the same set of winning coalitions, either value could be used to define the game. Since 44 cannot be attained, 45 was used.

7.1.3 *Spain joins the Union, or first problems with extending the game.* When Spain and Portugal joined the Union (1986), Portugal was added to the group of 5-vote states. Spain's population (38.6 million) was then much closer to France's (54.1) than to the Netherlands' (14.6), so the club of the most powerful states could admit Spain for 5th member. Otherwise one had to add a step in the ladder between the four big and four middle size states. Without changing the collection of weights, this could have only been done by assigning to Spain 6, 7, 8, or 9 votes. If two extreme numbers are discarded, one has to choose between 7 and 8. The choice of the greater number can of course be interpreted as Spain's political victory, but this country's ability to win negotiation games is not the only plausible explanation of why 8 was chosen.

Let us take the population of the largest country in each of 5 groups which make up the set of 12 states for the basis for determining the common weight for the group. Let 10 be the maximum weight, or the number of nominal votes granted to members of the top group. Table 5 shows two ways of assigning integer weights to the remaining groups. The first method is based on the postulate that the proportions of weights should be as close as possible to the respective population ratios. For example, since the population of Spain (38.6 million) was then 63% of the population of West Germany (Bundesrepublik had some 61.0 million citizen before absorbing DDR), Spain should obtain 10 times .63, or 6.3≈6 nominal votes (if the group mean were taken to represent the group, the ratio would equal .67, which translates to 7 votes). Under the second method, which uses square roots of the populations, the ratio equals 6.21/7.81=.795, which yields 8 votes for Spain.

Table 5. Two methods of determining integer weights from the population data

Group	Country	Pop.	Ratio	Pop. wght	Sqrt pop.	Ratio	Sqrt wght
1	Germany	61.0	1.00	10	7.81	1.00	10
2	Spain	38.6	.63	6	6.21	.80	8
3	Netherlands	14.6	.24	2	3.82	.49	5
4	Denmark	5.1	.08	1	2.26	.29	3
5	Luxembourg	.4	.01	0	.63	.08	1

As shown in Table 5, the weights which were actually assigned to 12 states may have been calculated by means of the square root method, the case of Luxembourg being the only exception.

Theoretical reasons for the use of square root weights will be discussed in Section 8 of this paper. Now let us try to guess how the top players might approach the problem of determining the quota for the Twelve. Once they agreed to give to Spain 8 votes, it seemed unlikely that they would make further concessions. Therefore, the Big Four should have demanded that r be equal to 19 or 20 (q=58 or 57) in order both to exclude Spain from blocking in pairs and to guarantee this privilege to themselves. Quite unexpectedly, as if community spirit overcame greed for power, the quota was set at (4·10+8+5)+1=54, or the minimum number such that any winning coalition must consist of 7 states. As a consequence, since r=23 for q=54, the minimum size of a 'blocking minority' was raised to 3.

The blocking structures in the games designed for the Nine, Ten, and Twelve are displayed in Table 4. All of them are multilevel, but none of them is regular. The lack of regularity can be explained in two alternative ways. Political decision-makers may have been interested only in the lowest level of the blocking structure. In estimating blocking power, they simply didn't take into account larger coalitions. But it may well be that irregularity was consciously approved as a way to ensure balance of blocking power between stronger and weaker players. The latter may have been granted more opportunities to block in larger coalitions to compensate for being denied access to smallest size blocking coalitions.

Voting games can be classified into four 4 types with respect to the shape of the premier league. The premier league can be *exclusive* or *inclusive*, and *hierarchical* or *egalitarian*. For the EU games, exclusiveness can be operationally defined by the condition that the premier league contains at most 1/3 of all players. All configurations, except the inclusive-egalitarian type, occur in the history of EU games. The historical importance of the game designed for the Twelve consists in the transition from the exclusive-egalitarian type to the inclusive-hierarchical type. In the Twelve, the premier league consists of 11 players (only Luxembourg cannot participate in a blocking three), but the distribution of the number of blocking threes is uneven.

7.2 *The case of EU*-15, *or a smart use of the theory of blocking power*

Table 6 shows the blocking structures in 4 voting games which were probably considered in designing a voting system for the Fifteen. The sum of the weights of 7 largest countries now equals 58. Therefore, to avoid constructing a double majority voting system (in our terminology, intersection of two weighted voting games), one should try quotas from 59 upwards. The quota actually used was 62.

Why the smallest relevant quota was not used? Two columns under G_1 show the numbers of minimal blocking coalitions of size 3 and 4 containing each of 15 states. Only the Big Four is granted the right to block in threes. Games G_2, G_3, and G_4 extend this right to Spain. Why games G_2 and G_3 were rejected and G_4 was accepted? All three games have the same set of blocking threes. The differences appear on the second of two levels of the blocking structure, that of blocking fours. Notice that G_2 and G_3 unlike G_4 do not meet the condition of regularity to the disadvantage of Spain. What a mathematical-political scientist cannot tell without consulting political actors involved is only whether a smart expert working for Spain outwitted the Big Four, or the strongest players agreed to admit Spain to the premier league as well as to give Spain the right place on the second level of the blocking structure.

Table 6. Blocking structures in 4 possible voting games for EU-15

State	Wght	G_1:q_1=59		G_2: q_2=60		G_3: q_3=61		G_4:q_4=62	
1. Germany	10	3	72	6	87	6	125	6	153
2. France	10	3	72	6	87	6	125	6	153
3. UK	10	3	72	6	87	6	125	6	153
4. Italy	10	3	72	6	87	6	125	6	153
5. Spain	8	0	60	6	24	6	56	6	108
6. Netherlands	5	0	36	0	60	0	74	0	86
7. Greece	5	0	36	0	60	0	74	0	86
8. Portugal	5	0	36	0	60	0	74	0	86
9. Belgium	5	0	36	0	60	0	74	0	86
10. Sweden	4	0	30	0	30	0	64	0	74
11. Austria	4	0	30	0	30	0	64	0	74
12. Denmark	3	0	6	0	24	0	36	0	64
13. Finland	3	0	6	0	24	0	36	0	64
14. Ireland	3	0	6	0	24	0	36	0	64
15. Luxembourg	2	0	6	0	0	0	24	0	36

The history of designing voting games for the EU Council abounds in dramatic turns, but it is by no means so absurd as it always appeared to the mainstream analysts who were able to notice only that the relative quota varied around 71%. In the language of our non-classical theory, we can say that Spain's action brought about a change of the shape of the premier league from the inclusive-hierarchical type to exclusive-egalitarian type.

The counteraction of the smaller states, which lost the privilege of blocking in threes, resulted in what became known as the *Ioannina compromise*, or the provision that the old blocking threshold (23 votes) will remain in use, but only for meta-deciding on whether a proper decision arrived at by applying the new voting system shall take effect immediately or shall be suspended for some 'reasonable' time to allow for more discussion and possible revision of the original act.

7.3. *From the Nice treaty to current controversies*

7.3.1 *Nice weights.* A historical-theoretical analysis of the Nice voting system must begin from an attempt to explain why those who designed the voting system for EU-27 found it necessary to discontinue the practice of assigning weights to new members by appending them to already existing groups (in the previous enlargement, Finland joined Denmark and Ireland) or occasionally creating a new group (Sweden and Austria were given 4 votes each, the only integer between 3 and 5). Such a procedure could have been applied to 12 states, but then 3 very small countries, Latvia, Slovenia, and Estonia, for the lack of intermediate integer between 2 and 3 had to join the last group (with Cyprus, Luxembourg and Malta) or the second one from the bottom made up of countries with population ranging from 3 to 5 million. However, a more serious problem was the necessity to place too many countries with too widely varying population numbers in the group whose original members were only Belgium and the Netherlands. The discussion of this problem at the negotiation table might have inspired Germany (now having over 80 million citizens) to demand a change in the allocation of weights within the strongest group as well.

The system of weights based on 10 as the maximum value was invented for 9 states, so it was quite natural to take 30 as the baseline weight for a voting system for the Council of 27 members. If the parity principle is retained and old proportions of weights are preserved, then the allocation of weights on the top would be: {Germany, France, UK, Italy}–30 votes, {Spain, Poland}–24 votes, and {Romania, Netherlands}–15 votes. Now I should warn the reader that this part of my analysis is highly speculative. However, when negotiation records are not available, any reasonable conjecture as to the origin of strange numbers 29 and 27 occurring on the top of the list of Nice weights is better than no explanation at all. If you find it likely that Poland and Spain were given 3 extra votes each, and these votes were gained by taking away 1 vote from the numbers originally assigned to 4 stronger and to 2 weaker players, then there still remains the puzzle of why the constructors of the game for EU-27 agreed to distort the square root proportions.

7.3.2 *Three variants of the Nice game.* The next step in constructing the game for EU-27 was the choice of a quota. As previously, largest weights (now of 13 players) were added up to determine q=257+1=258.The respective r=345–258+1=88 turned out to be the smallest blocking threshold under which 4 became the minimum size of a blocking coalition.

The first variant of the Nice game defined in such a way has one-level blocking structure. Players 1–27 participate in 411 blocking fours with the following frequencies: 1–4 (214), 5–6 (196), 7–21 (20), 22–27 (16). The premier league containing all players consists of two groups of unequal size. While within-group differences are very small, the Big Six has gained enormous power advantage over the remaining 21 states. Who was more disappointed at the 'two-class model' of political structure planned for EU-27? The Big Four, which had to acknowledge growing aspirations of the Semi-big Two, or the Twenty One fearing the domination of the Big Six? As it were, in the next phase of the negotiations, the blocking threshold was changed. We would not learn that q=258 had been considered, hadn't the negotiators left this number in the text of the treaty along with the new blocking threshold. Interestingly, contrary to the custom of concealing the real ‘methodology’ behind legal formulations, the number 91 appeared explicitly in the treaty as the minimum size of a ‘blocking minority.’ Since for r=91 we have q=255, there appeared an inconsistency which was noticed by classical theorists who were first to analyze the Nice game (Felsenthal and Machover 2001).

It is not clear why the old blocking threshold was replaced by 91 rather than another number greater than 88. 91 is perhaps a trace of the original hypothetical assignment of weights. If four largest states are assigned 30 votes each, then 91 is the smallest threshold under which blocking in threes becomes impossible.

Since the winning threshold of 255 votes can be attained by few coalitions having less than 14 members, the negotiators found a remedy in adding the requirement that a ‘qualified majority’ must comprise at least 50% of the members of the Council. That's how was born the second variant of the Nice game – the first ‘double majority’ voting system in the history of the EU. The premier league now included all players except Malta. The distribution of 315 blocking fours (1–4 (170), 5–6 (140), 7–21 (20), 17–21 (16), 22–26 (4)) differs from the one obtained for the first variant only with greater inequality within the Big Six.

That is not the end of the story. The third, final variant, or a ‘triple majority game’ was produced by putting into the already complicated system another component, the qualified majority game, now with population weights and relative quota 62%. The population component was added to give to the Big Four the privilege of blocking in threes. Spain, owing to its enormous population growth, joined the premier league soon after the Nice treaty was signed (2001). The premier league, which since then until today consists of 5 players, is the first example in the history of the EU of combining high degree of exclusiveness with within-group hierarchy.

7.3.3 *The Convention game and the EU Constitution game.* Soon after the final version of Nice treaty was signed (February 2001) a mysterious body known as the Convention designed a new voting system to be included in a new treaty that was to replace the Nice treaty. The game with ‘political’ weights was discarded to simplify the hybrid system rightly believed to be too complicated. In addition, in the double majority game which was left, the quota in the population component was lowered from 62% to 60%. The effect of the latter decision was the exclusion of Spain from the premier league, the second – making Germany's advantage over France, UK and Italy more permanent. Indeed, the Three's share of the total population of EU-27 was approaching 37%, a value too close to the threshold of 38%.

In Poland, the change of the population quota went unnoticed. Poland's angry reply (*Niza o muerte*) to the Convention's proposal expressed the disappointment that the idea of the Big Six (easy to read from the numbers 29, 27, 14 alone) was put to death soon after its birth. In addition, the draft of the Constitution treaty became known soon after the accession referendum decisively won by pro-European forces. Poland's relative *political* weight 27/345=7.8% roughly coincides with the share of the total EU-27 population. If voting power is equated with relative weight, then Poland has an equally strong position in both components of the Nice game. Why, therefore, not to get rid of the arbitrary political weights and keep the population component only? That's how the naive theory of voting power could have been employed to persuade the Poles to approve of the Constitution voting system. However, when Polish classical theorists calculated the Banzhaf index for the Nice and Convention game and showed the bar charts to the public, over 100 eminent scholars and intellectuals signed (November 17, 2003) an appeal to stand by the Nice Treaty.

Looking at the bar charts, I was surprised, above all, at the fact that the bar illustrating the value of the Banzhaf index for Germany in the Convention game stood out high above the bars for France, UK and Italy. When I took part in a conference of Polish experts (Kraków, March 29, 2004; see the blog-like part of my website), I still believed that politicians rely on classical indices. When the EU leaders decided (by the end of 2003) to restrict the search for a compromise to the family of double majority games with the population component, I examined all games of the kind with the aim to find one lying in the middle between the Nice game and the Convention game with respect to the Banzhaf index values. When I saw that the 'Jagiellonian game' (then recommended by Słomczyński and Życzkowski) was close to the *compromise* solution in this sense, I joined the international group of signatories of the open *Letter to the Governments of EU Member States* (June 2004).

Table 7. Blocking structures in the double majority game and the Constitution game

EU-27 member states	Double majority Game			Constitution Game	
	$b_{m,3}(i)$	$b_{m,4}(i)$	$b_{m,5}(i)$	$b_{m,4}(i)$	$\gamma(i)$
1. Germany	9	30	363	229	79.8
2. France	5	36	353	149	51.9
3. UK	5	29	287	142	49.5
4. Italy	5	27	238	140	48.8
5. Spain	3	37	141	107	37.3
6. Poland	3	17	226	87	30.3
7. Romania	0	28	125	38	13.2
8. Netherlands	0	11	172	21	7.3
9. Greece	0	8	101	18	6.3
10. Portugal	0	7	101	17	5.9
11. Belgium	0	7	101	17	5.9
12. Czech R.	0	7	101	17	5.9
13. Hungary	0	5	119	15	5.2
14. Sweden	0	5	100	15	5.2
15. Austria	0	3	114	13	4.5
16. Bulgaria	0	3	104	13	4.5
17. Denmark	0	0	84	10	3.5
18. Slovakia	0	0	84	10	3.5
19. Finland	0	0	84	10	3.5
20. Ireland	0	0	58	10	3.5
21. Lithuania	0	0	52	10	3.5
22. Latvia	0	0	36	10	3.5
23. Slovenia	0	0	31	10	3.5
24. Estonia	0	0	22	10	3.5
25. Cyprus	0	0	16	10	3.5
26. Luxembourg	0	0	11	10	3.5
27. Malta	0	0	11	10	3.5
	10	65	647	287	

Most experts expected that the winning threshold in the population game would be lower than that proposed by the Convention. Actually, June 18, 2004, the EU summit set the quota at 65%, which might satisfy the Three as well the Semi-big Two. However, the distribution of the number of blocking threes across 6 teams which form an exclusive premier league turned out extremely hierarchical. Maybe the historians will find out some day who was first to notice that Germany is a member of 9 out 10 blocking threes, by far surpassing other largest countries in this respect. As a consequence, the Inter-Governmental Conference ruled in the last minute that the following clause will be put into article I-25 of the Constitution treaty: *A blocking minority must include at least four Council members, failing which the qualified majority shall be deemed attained*.

Table 7 shows that the double majority game has a highly irregular blocking structure. The ban on blocking in threes results in guaranteeing to every player an opportunity to block in fours, the one level blocking structure is regular, but the distribution of the number of blocking fours is very uneven.

The conclusion which follows from the historical analysis is that the operationalization of blocking power offered in this paper adequately renders the approach of the 'natives.' When I did my first mathematical-political analyses, I had not yet realized that there is a *scientific* alternative to the mainstream approach to voting power. The observation that classical theorists more worry about power inequality within the top group than its members made me hypothesize that the Banzhaf index values may be too weakly correlated with the values of a power measure used by the players themselves. In light of my current research, such a conjecture must be rejected. You will detect Germany's dominating position in the Constitution game, which now can be called the Lisbon game, whether you calculate the Banzhaf index or the relative frequency of blocking fours. If you use the measure γ, you will notice that the largest difference in power between two states occupying neighbor positions in the power ordering occurs between Germany and France. Paradoxically, it was Poland not France that objected to the Constitution voting system.

8. Blocking power in square root games

8.1 *The rationale for the use of square root weights*

8.1.1 *The two-tier voting game.* Under the naive approach to voting power, the replacement of raw weights with their square roots can only be motivated by the intention to *flatten* the distribution of relative weights. To produce this effect, considered desirable by the EU rank and file members, but strongly contested by the top players, population numbers can be mapped into weights in many ways. The rationale for the choice of the square root function was given long time ago by Lionel Penrose (1946). He not only laid the foundations of the *classical* theory of voting power, but proposed a *two-tier voting game* as a formal model of *indirect* voting. His model is presented below after Felsenthal and Machover (1998).

Assume that the set $\mathcal{N}$ of voters (let us call them *citizens*) consists of m pairwise disjoint sets (referred to as *constituencies*) $\mathcal{N}_1,..., \mathcal{N}_m$. When a proposal is put to the vote, citizens decide first independently in each constituency $\mathcal{N}_i$ by means of a voting game with some $\mathcal{W}_i$ as a set of winning coalitions. If the set C_i of members of $\mathcal{N}_i$ who voted for the proposal is a 'locally' winning coalition ($C_i\in\mathcal{W}_i$), then the *delegate* d_i of ith constituency is bound to vote 'yes' when the *council* of delegates $\mathcal{M}=\{d_1,...,d_m\}$ is to finally decide on the issue, using its own voting rules formalized as $(\mathcal{M},\mathcal{V})$. The set $\mathcal{W}$ of winning coalitions in the *two-tier voting game* over $\mathcal{N}$ consists, by definition, of all subsets C of $\mathcal{N}$ such that the delegates who represent the constituencies $\mathcal{N}_i$ in which $C_i=C\cap\mathcal{N}_i$ is a winning coalition form a winning coalition in the game $(\mathcal{M},\mathcal{V})$. Formally, $C\in\mathcal{W}$ if and only if $A(C)\in\mathcal{V}$, where $A(C)=\{d_i: C_i\in\mathcal{W}_i\}$.

8.1.2 *Square root laws*. Penrose studied two-tier voting games which have 1 citizen–1 vote simple majority games on the bottom and a weighted voting game on the top. In a 1 *voter – 1 vote simple majority game*, a subset C_i of $\mathcal{N}_i$ is a winning coalition if and only if $|C|>n_i–|C|$ where $n_i=|\mathcal{N}_i|$. Every game of the kind is *symmetric* (any two players are structurally interchangeable), and so is the 1 voter–1 vote simple majority game with the set of players $\mathcal{N}=\mathcal{N}_1\cup...\cup\mathcal{N}_m$. The latter game is a model of *direct* democratic decision mechanism in assembly $\mathcal{N}$. Symmetry implies, in particular, that all players are granted *equal* theoretical influence on collective decisions. Penrose showed that it is possible to construct a system of *indirect* voting which approximately meets the same condition of equality. His theorem (Th. 3.4.3 in Felsenthal and Machover 1998: 66) states that if all n_i are sufficiently large, then the two-tier voting game of the form described above has an approximately even power distribution if and only if the power of each delegate in the game ($\mathcal{M},\mathcal{V}$) is roughly proportional to the square root of the size of his constituency, or equivalently, for any two delegates d_i and d_j, the ratio of their voting powers negligibly differs from $\sqrt{n_i}\big/\sqrt{n_j}$. Needless to say, the voting power of a citizen or delegate was quantified by Penrose by means of the number of winning coalitions containing a given player as a critical member.

Felsenthal and Machover (1998, p. 68) noticed that Penrose's theorem, usually referred to as the '(first) square-root law/rule' is often misstated by saying that all citizens have equal voting power in a two-tier game if the *weights* in the game ($\mathcal{M},\mathcal{V}$) are proportional to the square roots of n_i. In the correct statement, proportionality is required of the delegates' voting powers not weights. However, Słomczyński and Życzkowski (2006) showed that voting power can be made proportional to square root weight by setting the quota at a special value they called 'optimal.'

It seems unlikely that EU politicians would ever accept a *unique* scientific quota. They will not renounce the control over the quantity which under fixed weights determines the set of blocking coalitions. As regards scientific weights, there is more hope that their mathematical underpinning will not be disregarded.

To justify the use of $\sqrt{n_i}$ instead of n_i, one can also invoke the theorem known as the 'second square-root law/rule' (Felsenthal and Machover 1998: 72–78). To show what this theorem is about, I will try to explain – I hope, precisely enough for mathematical readers and intelligibly enough for non-mathematical ones – what is meant by the *mean majority deficit* of a two-tier voting game ($\mathcal{N},\mathcal{W}$). According to a theorem proven by Felsenthal and Machover (1998: 60, Th. 3.3.17), this quantity shows how much the power distribution in a given two-tier game differs from the even distribution in the 1 citizen–1 vote simple majority game with the same set of voters. To put it even less formally, the mean majority deficit allows us to assess how equitable is a given system of *indirect* voting by comparison with the standard system of *direct* voting based on the democratic principle that all citizens should have *equal* potential influence on collective decisions. Within this conceptual framework, one can pose the following interesting problem: given 1 citizen–1 vote simple majority games on the bottom of a two-tier voting game and a *weighted* voting game ($\mathcal{M},\mathcal{V}$) on the top, for which $\mathcal{V}$ the mean majority deficit is smallest. If the n_i are sufficiently large, then the solution to the problem is a general simple majority voting game in which the delegates' weights are proportional to square roots of the sizes of the respective constituencies.

The collection of winning coalitions in a *general simple majority voting game* ($\mathcal{M},\mathcal{U}$) with weights $u_1,...,u_m$ assigned to the members of $\mathcal{M}$ is defined by the condition: $A\in\mathcal{U}$ if and only if

$u(A)>u(M–A)$ where $u(A)$ is the sum of weights of members of A (in particular, if $u(i)=1$, for $i=1,...,m$, we get the 1 voter–1 vote simple majority game in which $u(A)=|A|$). A general *simple* majority game can be defined equivalently as the weighted voting game with weights $u_1,...,u_m$ and the quota equal to the minimum of $u(A)$ over all A such that $u(A)>½u(M)$.

Prior to considering practical applications of the square root laws, one must understand the difference between indirect representation of EU citizens in the Council of Ministers of EU by their governments and their indirect representation in the European Parliament by the sets of deputies elected in particular countries. Since the members of any parliamentary group are not bound to vote unanimously, national factions, unlike governments, must not be considered as players in a voting game. Therefore, there is no reason to assume in advance, as do naive EU democrats, that the number of indivisible votes in the Council and the number of individual votes in the EP should be related to a state's population by the same linear function.

The supporters of the EU Constitution voting system who point to its 'democratic' character should think over the following fictitious, yet by no means unrealistic story. Suppose that all citizens of the EU are asked the question: Do you agree to give more power in the Council to the governments representing states having each more than 10% of the EU total population? Imagine now that a (fictitious) social movement 'Europeans for Democratic Europe' launched a campaign following which the frequency of 'yes' in each of 4 largest states dropped to 70%, while in each of 23 smaller states at most 20% of all citizens responded positively to the slogan 'Give more power to the Four if you want them to help you more.' The citizens of the Big Four make up today 53.6% of the total EU population. Hence, the proposal would fail in a referendum, for it gained support of no more than $0.536·0.7+0.464·0.2=46.8\%$ of all EU citizens. What would be the outcome of an indirect voting? If all governments respect the results of the poll in their states, and the states' shares of the total EU population are taken as weights in the voting game to be played by the Council, then the proposal will be passed, even it is backed by a minority of EU citizens. If the general simple majority game with square root weights is used on the upper tier of a two-tier voting system, then the outcomes of direct and indirect voting are more likely to be identical. In our concrete case, the proposal, which was backed by 4 largest states only, will fail since the sum of square root weights assigned to them is 34.2% of the total. One can try, of course, other ways to bring back the power to the EU citizens (raising the winning threshold in the Council and/or adding a 1 delegate–1 vote game with a proper quota), but why not to use square root weights which provide the simplest solution to the problem.

8.1.3 *Which way forward and to where?* The EU member states still have retained much of their sovereignty, first of all, in the domain of international politics. For example, the Union can't start negotiations with an external nation without the consent of all its member states. They have the right of veto in this matter and occasionally use it as recently did Poland by refusing its mandate for negotiations with Russia. When the USA wants to conclude a new treaty with Mexico, Texas cannot block this initiative because the states which form one nation have irrevocably delegated all decisions in the area of foreign affairs to the federal government. With narrowing down the range of issues decided by the *consensus game*, the EU will also evolve toward a *federation* of states. A sign of the same trend is gradual increase of the minimum size of minimal blocking coalitions in voting games that are used to decide where unanimity is no longer required. When blocking is getting more and more difficult, the EU leaders will be more concerned about the measurement of winning power and should finally appreciate the virtues of the square root function which is the most natural way of assigning weights to the delegates of *varying in size* autonomous political communities united into a federation.

The use of square root weights, first suggested by Sweden in Nice, did not find support from those EU member states which have always fostered advanced integration of the Union. Paradoxically, Poland, known to have always objected to making the EU a federal superstate, was the country that fought (and lost) the ‘battle for the square root’ with France and Germany.

It is no less paradoxical that the square root mapping of the population structure into weights is still regarded by the EU establishment as a purely theoretical, magic or subversive idea. Actually, as I have shown in Section 7 of this paper, square root proportions between weights are found in the pre-Nice voting games, albeit it is doubtful if their constructors have consciously applied Penrose's theory. The Convention broke with this tradition by making the population weights (first introduced in the Nice treaty with the intention to ‘correct’ the power distribution) the basis the new voting system. The ‘demographic principle’ has still been stubbornly defended by the supporters of the Constitution treaty, even if their main argument has changed to: ‘Let's stay with the decision already made in order not to open the Pandora's box once again.’ Although the EU leaders were probably unaware of the hegemonic nature (revealed in this paper) of the new system, they might have feared the unknown, on the same basis as they got scared of the ‘square root revolution.’ As a consequence, the EU summit (2007) decided to stay until 2014 with the familiar Nice voting system to which even its opponents had already got used.

8.2. *The Jagiellonian game and the French game*

Why did the Big Four approve of such a ‘compromise’ probably sought by Poland from the outset? As we shall see, if they accepted square root weights, they could gain more blocking power than they had in the Nice game and no less than in the Constitution game. Their prejudice against square root weights probably stemmed from relying solely on naive theory of voting power. If relative square root weights were used instead of relative population weights, Spain and Poland's position on the power scale would come closer to Italy's. However, if ‘power’ means ‘blocking power’ measured by means of the number of smallest size minimal blocking coalitions containing a given player, then without specifying the quota one cannot tell how big is the advantage of the Four over the Two.

Poland risked a lot, when it came up with a proposal to use square root weights to design a new voting system for the EU Council. If Germany and France agreed to include the Polish proposal in the list of issues requiring more discussion before accepting the final version of the EU Reforming Treaty, Poland would have to respond with one's own concession, that is, to accept the partners' claims when it would come to negotiating a *politically* optimal quota.

Square root weights, represented as integers which add up to 345, can be used to design many voting games for the Council of Ministers of EU-27. To dispel German doubts as to the Polish initiative, I examined (Sozański 2007a) a voting game obtained by setting the quota at 246. This game has a two-level blocking structure with blocking fours on the lower level. The premier league consists of 16 countries (from Germany through Bulgaria) whose order with respect to the number of blocking fours reproduces the order of weights. If you compare this particular square root game with the double majority game (Table 7) which has the same players on the second level of the three-level irregular blocking structure, you will see that Germany's *will to power* could be fulfilled to an even greater degree if an appropriate quota were used together with ‘Polish weights.’ Unlike the original double majority game, the ‘German square root game’ has a regular blocking structure and need not be artificially corrected by adding the ban on blocking in threes.

Table 8. The blocking structure in two games with square root weights

EU-27 member states	Square root weights	Jagiellonian game q=213			French game q=255	
		$b_{m,5}(i)$	$b_{m,6}(i)$	$\gamma(i)$	$b_{m,4}(i)$	$\gamma(i)$
1. Germany	33	7	588	89.9	174	72.8
2. France	29	6	495	75.7	137	57.3
3. UK	28	6	466	71.3	123	51.5
4. Italy	28	6	466	71.3	123	51.5
5. Spain	24	4	355	54.2	94	39.3
6. Poland	22	4	255	39.1	74	31.0
7. Romania	17	1	181	27.5	27	11.3
8. Netherlands	15	1	151	23.0	20	8.4
9. Greece	12	0	100	15.1	17	7.1
10. Portugal	12	0	100	15.1	17	7.1
11. Belgium	12	0	100	15.1	17	7.1
12. Czech R.	12	0	100	15.1	17	7.1
13. Hungary	11	0	86	13.0	13	5.4
14. Sweden	11	0	86	13.0	13	5.4
15. Austria	10	0	71	10.7	12	5.0
16. Bulgaria	10	0	71	10.7	12	5.0
17. Denmark	8	0	43	6.5	10	4.2
18. Slovakia	8	0	43	6.5	10	4.2
19. Finland	8	0	43	6.5	10	4.2
20. Ireland	7	0	33	5.0	8	3.3
21. Lithuania	7	0	33	5.0	8	3.3
22. Latvia	5	0	18	2.7	4	1.7
23. Slovenia	5	0	18	2.7	4	1.7
24. Estonia	4	0	14	2.1	3	1.3
25. Cyprus	3	0	8	1.2	3	1.3
26. Luxembourg	2	0	3	0.5	3	1.3
27. Malta	2	0	3	0.5	3	1.3
	345	7	655		239	

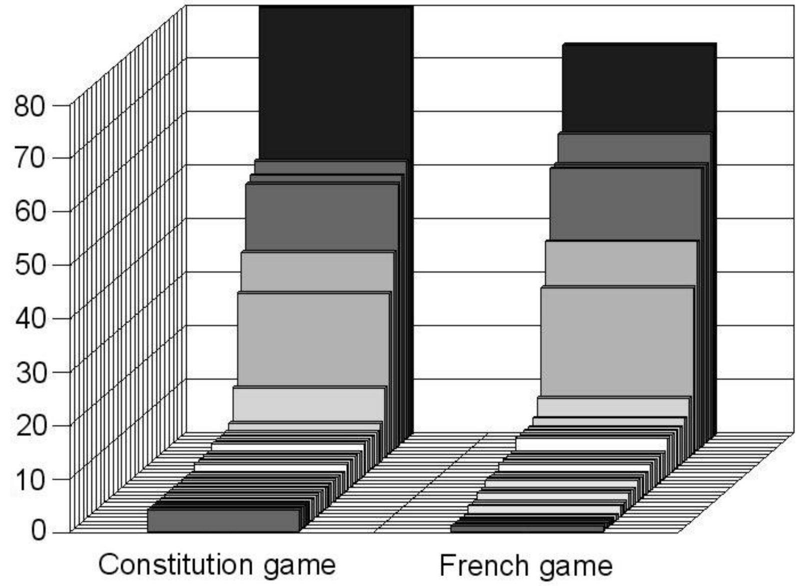
Let me examine here two other square root games. The first of them, the *Jagiellonian game*, is defined by setting the quota at 213 nominal votes, which is the integer counterpart of the 'optimal' relative quota of 61.6% recommended by Słomczyński and Życzkowski (2006). This game has a two-level regular blocking structure, with exclusive-hierarchical premier league made up of 8 largest states. Notice that Germany is a member of all 7 blocking fives and has considerable advantage over other countries on the level of minimal blocking sixes.

The choice of a quota is a suitable *technical means* to achieve a given *political end*. If you set the blocking threshold r at 91 (the minimum value such that blocking in threes becomes impossible), which yields q=255, you will not get the 'ugly' Nice game but a 'nice' game whose blocking structure very much resembles that of the Lisbon game except for reducing a little the advantage of Germany over France.

The chart given below illustrates the distributions of relative blocking power in the two games. In a voting game having one-level blocking structure with k_{min}=4, *relative* blocking power can be measured by means of index γ' defined by the formula $\gamma'(i)=b_{m,4}(i)/\sum b_{m,4}(j)$.

Those who postulate that the distribution of relative power be as similar as possible to the distribution of population shares should appreciate another property of the 'French square root game.' Although this game has been constructed with the use of square root weights, the values of γ' are very close to the shares of the total EU population.

Since $\sum b_{m,4}(j)=4b_{m,4}$, we can easily compute $\gamma'(i)$ from the data given in Table 8, by dividing $b_{m,4}(i)$ by $4b_{m,4}=4\cdot239=956$. For Germany and France we get $\gamma'(1)=174/956=18.2\%$, $\gamma'(2)=137/956=14.3\%$. These values are by 1.5% greater than the relative population weights 16.7% and 12.8%. For all other countries except Romania ($\gamma'(7)=2.8\%<4.4\%$) and the Netherlands ($\gamma'(8)=2.1\%<3.3\%$), the respective difference does not exceed 1.0%



Similar calculations can be done for the Lisbon game. France's relative blocking power (13.0%) in this game is only a bit greater than this country's share of the EU population (12.8%), whereas Germany's power excess over the population share (16.7%) now equals 3.2% ($\gamma'(1)=229/(4\cdot287)=19.9\%$). Romania and the Netherlands gain again a bit less relative blocking power than their population shares. For other countries, the absolute difference between the two values never exceeds 1.0%. Thus, Germany would be the only beneficiary of the Lisbon voting system if it took effect as of today. Fortunately, its implementation was put off for a sufficiently long time given to European politicians for reconsidering the ideas of European mathematicians.

9. Conclusions, or again on politics and mathematics

Sometimes mathematical theorizing is at odds with actual 'political engineering', as it happened to the mainstream theory of voting power. Nevertheless, the mathematicians will always have *knowledge advantage* over the politicians. They are able to grasp the latter's point of view and reorient their investigations accordingly. That's exactly what I did, having realized that EU leaders pay little attention to the values of classical indices of winning power, trying instead to maximize their states' *blocking* power.

There is another asymmetry. The politicians have always had *power advantage* over the mathematicians. The former can say to the latter: we do appreciate your efforts to enlighten us, but the designing of decision rules for *practical* use must remain our exclusive right. I'm not going to deny this prerogative to the EU leaders, yet in their own interest they shouldn't ignore the voice of scientists who can often reveal some *unintended consequences* of political decisions, such as potential *instability* of the distribution of blocking power in voting games in which the rules, for being based on external data (e.g., *current* demographic composition), may appear sensitive to small unpredictable changes in the input.

Consider the following example. If Croatia with its population equal to 4443 thousands joined the EU today, then Germany, France and Spain would still form a 'blocking minority' because their total population would be 38.02% of the total population of EU-28. However, since the excess over the threshold of 38% is negligible, a slight change in the population structure might deprive the Berlin-Paris-Madrid axis of its power to block any decision of the Council. Thus, the Nice voting system, being partially based on variable population weights, prompts Spain to oppose further enlargements, as it is the only way to save its privilege of blocking in threes.

What the politicians must learn, first of all, if they are to benefit from cooperation with experts in the mathematical theory of voting games, is the distinction between strictly political issues and technical problems. The task of the politicians is to work out an agreement as to political requirements that a voting system should answer. Whether the EU leaders choose to restore the parity principle or they finally endorse Germany's claim to power advantage over other largest EU member states is a *political* issue. *How* to design a voting game which generates a given power hierarchy and meets certain formal conditions, conditions which may be no less politically important, such as regularity of the blocking structure, is a *technical* problem. The 'French game' I have constructed with the use of square root weights with the aim to bring down Germany's power advantage over France to a reasonable size shows that *independent* experts can solve technical problems whether they do or do not approve of political ends which determine to range of solutions they have to consider.

That's how I perceive the task of an expert, but I realize that the expectations of those who seek advice from him may have a stronger effect on how the expert plays his *social role* than the expert's own role conception. The men and women of power usually expect from the men and women of knowledge to be instructed solely as to the *means*. They may welcome being enlightened in the matter of *ends* as well when they are unable to define their objectives sufficiently clearly, as is the case with specifying a measure of voting power to be maximized. However, the politicians may well reject any help from the experts for fear that the latter might misrepresent their intentions or try to promote their own conceptions or evaluations. Is this fear justified? As a matter of fact, academics love to persuade politicians to adopt both certain general principles and concrete policies they believe to be grounded on scientific truths. So did I when I signed the *Letter to the Governments* recommending the use of square root weights in designing a voting system for the EU Council. Three years later two signatories of that letter, professors Jesús Mario Bilbao (University of Sevilla) and Karol Życzkowski (Jagiellonian University) were given an opportunity to defend Penrose's theory once again at the debate which took place May 23, 2007 at the European Policy Center in Brussels. I will close my paper with my own answer to the question 'Which way forward?' which was asked at that debate, namely, let me state the following suggestions (which I first formulated in a letter that I sent to my colleagues before the debate) for the EU leaders and *their* experts.

- If you do not accept the *classical theory of voting power*, you should appreciate square roots weights as a convenient *means* for designing voting games according to the *theory of blocking power* I developed to help myself and you to understand what you are doing.
- Represent square root weights in a fixed integer form in order to be able to determine exact numbers of small blocking coalitions. The integer weights may add up to the number given in the Nice treaty (345) or a comparable number.
- Simplify the allocation of weights by dividing the set of players into groups that are assigned the same weight. Here political decisions have to be made. If you choose to finally part with the *parity principle* in relation to the Big Four, tell it overtly to the world instead of hypocritically invoking the principle of democratic representation to justify the decision to make Germany the strongest player.
- For given political weights obtained by adjusting original square root weights, try various quotas until you manage to construct a voting game with blocking structure that will be both regular and politically acceptable for all EU member states.
- If you can't find a better solution, implement the Jagiellonian voting game which is well constructed and should be politically acceptable, too, for it gives enough blocking power to Germany – the leader of the 'premier league' in this game. If you believe that the EU needs further integration and reaching positive decisions should be made easier, then you should not object to raising from 4 to 5 the minimum size of any blocking minority.
- You can add the 1 state–1 vote game with a proper quota (say, 15 countries) if the winning threshold you have chosen for the game with square root weights can be attained by some coalitions formed by less than a simple majority of states. The set of *small* blocking coalitions will not change, but smaller countries will be more comfortable with a system whose egalitarian component has a stronger effect on forming *winning* coalitions.

The treaty reforming the EU institutional structure was signed in Lisbon 13 December, 2007 with 1 January 2009 as intended date of entry into force. However, once the treaty has not yet been ratified by all member states, the Nice treaty remains in force. Thus, there is again some time for reconsidering the task of constructing a voting game that the EU Council will use to decide on the issues excluded from the scope of applicability of the consensus game. The decision on which issues are to be decided in either way is much more important for the future of the EU, yet paradoxically the latter decision appeared much easier to be made by the politicians. Because of a purely political nature of the task? If so, there is still in the EU a good deal of political *will* to proceed with institutional integration. But when both political and technical problems have to be solved, as is the case with defining a nonconsensual voting system, what is needed apart from good will is *reason* armed with a robust theory. The aim of this paper was to meet this need some men and women of politics occasionally feel.

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